Exterior ballistics

John Forsyth Meigs, Royal Rodney Ingersoll

Ballistics, 1883

TD

Meigs

not in RW 12/3/29

EXTERIOR BALLISTICS;

PREPARED AND ARRANGED

FOR THE USE OF CADETS AT THE U. S. NAVAL ACADEMY,

BY

LIEUTENANT J. F. MEIGS, U. S. N.

AND

LIEUTENANT R. R. INGERSOLL, U. S. N.

1885

CONTENTS.

PREFACE.

The methods of solving problems in Ballistics here given are substantially due to Professors Francis Bashforth and W. D. Niven; to the former, by his admirable series of experiments and methods of reducing the same, belongs the honor of having raised Exterior Ballistics to the dignity of a science.

In the preparation of the work, Sladen's Principles of Gunnery and Mackinlay's Text-Book of Gunnery have been freely consulted; the excellent Traité de Balistique Rationnelle, par J. Baills, and Didion's Traité de Balistique, have also been of great service.

CHAPTER I.

Ballistics is the science of the motion of projectiles, and Exterior Ballistics is that branch of the science which is concerned with their motion when outside the gun.

As certain terms will be continually used in the discussion of this subject, their definitions are here given:

The line of sight is a straight line passing through the two sight points; in the act of firing, it also includes the target.

The line of departure is the line in which the projectile is moving when it leaves the gun; it is, therefore, the tangent at the muzzle of the gun to the curve described by the projectile.

The axis of the bore is its geometrical axis, and is the line along which the centre of mass of the projectile should move while in the bore.

The axis of the trunnions is their common geometrical axis, and in recent guns it intersects the axis of the bore at right angles.

The angle of elevation is the angle included between the line of sight and the axis of the bore.

The angle of projection is the angle of elevation corrected for jump, which is the vertical angle which the axis of the bore describes in the act of firing. The latter angle is caused by straining and slackness of the gun and its carriage, and must always be found by experiment (*Text-Book of Ordnance and Gunnery*, p. 149).

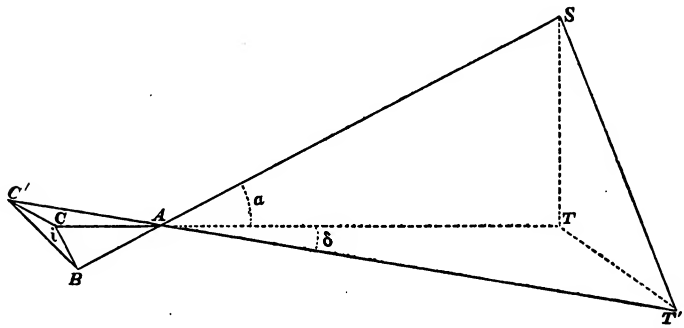
The angle of sight is the angle included between a line passing through the gun and target and the horizontal plane.

The angle of departure is the sum of the angles of projection and of sight; or is the angle included between the horizontal plane and the line of departure.

The theory of the sight bar, which is essential to the correct understanding of the science of ballistics, will also be presented here. There are several methods of laying guns so that they will hit any desired point, but in our navy one way is adopted to the almost entire exclusion of all others. It consists in so marking the sight bar

that, the distance of the target being known, the bar may be set; and then, when the line of sight includes the target, the axis of the bore is set to hit the target. The graduation of the bar in arc, and laying the gun by spirit level, may be different things from this. In all investigations concerning the sight bar, it is permissible to neglect the dimensions of the gun when compared with the range of the projectile and its deflection (the distance it goes to the right or left); the results thus reached will, for practical purposes, be correct. For example: we may consider that the sights, when the bar is down, are in the axis of the bore, and may consider the range to be measured from the front, or rear sight, indifferently.

If a sight bar be correctly marked to hit a point at some assigned distance from the gun, it is evident that this mark will not enable us to hit whatever be the vertical position of the point; in other words, we cannot hit any desired point of a vertical circle described about the gun with the range as radius; for the present, then, the position of the point to be hit will be restricted to the horizontal plane through the gun.



When a sight bar is to be marked, the given quantities are, the distance between sights, $AB\ (=l)$ in the figure; the angle of elevation, SAT or CAB; the range, AT; and the deflection, TT'. In the figure let BAS represent the axis of the bore prolonged, and CAT, in the same vertical plane, the line of sight, A and C being the two sight points and CB or $C'B$ the sight bar. Suppose that the projectile falls at T', having been deviated from the plane ATS by TT'; if then we draw $T'AC'$, and from the point C draw CC' at right angles to the plane ATS; then, if $C'AT'$ had been the line of sight, the point seen would have been the point struck. C' is consequently the proper position for the sight notch.

Let $CC' = d$, $BC = h$, $BC' = H$; then, since in most guns the three faces which we see of the tetrahedron $ABCC'$ are right-angled at B, C, and C', we have

$$\tan \alpha = \frac{BC}{BA} = \frac{h}{l}, \qquad \therefore\ h = l \tan \alpha.$$

$$\tan i = \frac{CC'}{BC} = \frac{d}{h}, \qquad \therefore\ d = l \tan \alpha \tan i.$$

$$\tan \delta = \frac{CC'}{CA} = \frac{d}{l \sec \alpha}, \qquad \tan i = \frac{\tan \delta}{\sin \alpha} = \frac{TT'}{AT} \operatorname{cosec} \alpha.$$

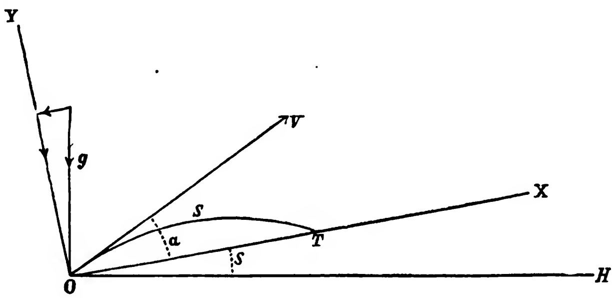
$$H = h \sec i.$$

To recollect the above figure, it is convenient to remark that it consists of two triangular pyramids with their vertices coincident, formed by the lines CT, $C'T'$, BS; and that the planes of their bases, when prolonged downwards, will intersect at the angle α.

CHAPTER II.

The Motion of Projectiles in a Non-Resisting Medium.

The resistance of the air to the motion of projectiles moving at ordinary velocities is so great that numerical results reached by considering their motion unresisted can seldom be applied in practice, but a correct understanding of the problems which arise is greatly aided by the facility acquired by the study of this comparatively simple case. It will in all cases be assumed that the only force acting upon the projectile during flight is that of gravity, and that this acts parallel to the vertical direction at the gun. The motion of the centre of mass of the projectile will therefore be confined to the vertical plane containing the gun and target.



Take the axis of X through the gun at O and target at T, and that of Y at right angles to it, as shown in the diagram; let OH be the trace of the horizontal plane through the gun. Then, in the figure, OV is the direction of the axis of the bore, OT the line of sight, α the angle of projection and s the angle of sight. The equations of motion are evidently, in the hypothesis assumed,

$$\left.\begin{aligned}\frac{d^2x}{dt^2}&=-g.\sin s\\ \frac{d^2y}{dt^2}&=-g.\cos s\end{aligned}\right\}\qquad(1)$$

Integrating and determining the constants by assuming that the shot is projected with a velocity V in the direction making an angle α with

the axis of X, and taking the origin of time at the origin of motion, we have

$$\left.\begin{aligned}\frac{dx}{dt} &= -gt.\sin s + V\cos a\\ \frac{dy}{dt} &= -gt.\cos s + V\sin a\end{aligned}\right\} \quad (2).$$

Integrating again, we have, since x, y, and t are zero together,

$$\left.\begin{aligned}x &= Vt\cos a - \tfrac{1}{2}gt^2.\sin s\\ y &= Vt\sin a - \tfrac{1}{2}gt^2.\cos s\end{aligned}\right\} \quad (3)$$

Multiplying the 1st and 2d of (3) by $\cos s$ and $\sin s$ respectively, and subtracting, we find for t,

$$t = \frac{x.\cos s - y.\sin s}{V\cos(a+s)}; \quad (4)$$

and by substituting this value in the 1st of (3),

$$x = \cos a.\frac{x\cos s - y\sin s}{\cos(a+s)} - \frac{g.\sin s}{2}\left[\frac{x.\cos s - y.\sin s}{V\cos(a+s)}\right]^2. \quad (5)$$

In this expression, putting $y = 0$, we find $x = 0$, and

$$x = \text{range} = \frac{2V^2}{g}.\frac{\sin a.\cos(a+s)}{\cos^2 s}. \quad (6)$$

The equation to the curve described by the projectile, with the assumed origin and axes, is (5); it is evidently a parabola. The time of flight and range for any values of V, a, and s, may be found from (4) and (6). The formulae are however greatly simplified by assuming that the gun and target are in the same horizontal plane; we have then, putting $s = 0$,

$$\left.\begin{aligned}x &= Vt.\cos a\\ y &= Vt\ \sin a - \tfrac{1}{2}gt^2\end{aligned}\right\}, \quad (7)$$

whence the equation to the trajectory is

$$y = x.\tan a - \frac{gx^2}{2V^2\cos^2 a}. \quad (8)$$

It is to be observed, however, that this is only a particular case. Let R = range on the horizontal plane through the gun, then from (8),

$$R = \frac{V^2}{g}.\sin 2a; \quad (9)$$

therefore, for any given value of V, the range is a maximum when $\alpha = 45°$. Equation (9) also shows that there are two values of α which are the complements of each other with which the point may be hit. We also have from the 1st of (7), and obviously, since the velocity in X is constant when $s = 0$,

$$T = \text{time of flight} = \frac{R}{V.\cos\alpha}. \tag{10}$$

For the inclination of the curve at any point,

$$\frac{dy}{dx} = \tan\alpha - \frac{gx}{V^2.\cos^2\alpha};$$

by putting $\frac{dy}{dx} = 0$ in this equation, we find the abscissae of the highest point of the curve; and by putting $x = R$, we find, calling β the angle between the tangent to the curve and the axis of X at that point, or the angle of fall,

$$\tan\beta = -\tan\alpha. \tag{11}$$

It is evident that the constants V and α may be so determined in (8) as to pass the trajectory through two points whose coordinates are given with reference to the gun. Let the points be $(a_1, b_1,)$ and $(a_2, b_2,)$: then we have from (8),

$$\left.\begin{aligned} b_1 &= b_1 \tan\alpha - \frac{g.a_1^2}{2V^2.\cos^2\alpha} \\ b_2 &= a_2.\tan\alpha - \frac{ga_2^2}{2V^2.\cos^2\alpha} \end{aligned}\right\}.$$

Therefore,

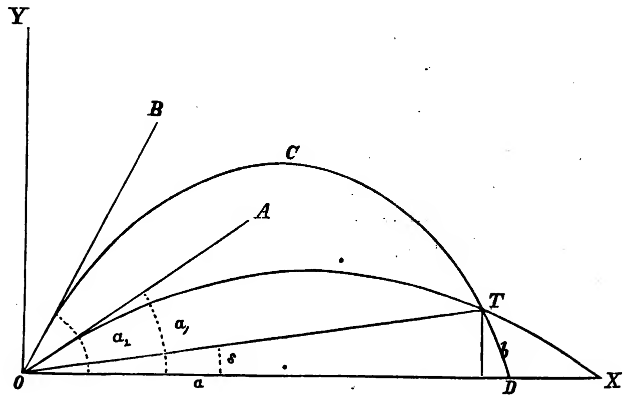
$$\left.\begin{aligned} \tan\alpha - \frac{b_1}{a_1} &= \frac{ga_1}{2V^2.\cos^2\alpha} \\ \tan\alpha - \frac{b_2}{a_2} &= \frac{ga_2}{2V^2\cos^2\alpha} \end{aligned}\right\}.$$

Dividing these, member by member, and writing $\frac{b_1}{a_1} = \tan s_1$ and $\frac{b_2}{a_2} = \tan s_2$, we have

$$\tan\alpha = \frac{a_2.\tan s_1 - a_1\tan s_2}{a_2 - a_1}. \tag{12}$$

Also,

$$V = \frac{1}{\cos\alpha}\sqrt{\frac{g}{2}.\frac{a_2 - a_1}{\tan s_1 - \tan s_2}}. \tag{13}$$



The following theorem is also instructive; it is taken from Didion's *Traité de Balistique.* In the figure, let a, b, be the coordinates of the target it is desired to strike, then from (8), since $V^2 = 2gh$, where V is the velocity due to the height h, we have

$$b = a \tan \alpha - \frac{a^2}{4h}(1 + \tan^2 \alpha).$$

Therefore,

$$\tan^2 \alpha - \frac{4h}{a} \tan \alpha + \frac{4hb}{a^2} + 1 = 0;$$

the two roots of this equation are given by

$$\left.\begin{aligned} \tan \alpha_1 + \tan \alpha_2 &= \frac{4h}{a} \\ \tan \alpha_1 . \tan \alpha_2 &= \frac{4hb}{a^2} + 1 \end{aligned}\right\}$$

$$\frac{\tan \alpha_1 + \tan \alpha_2}{1 - \tan \alpha_1 \tan \alpha_2} = \tan (\alpha_1 + \alpha_2) = \frac{1}{-\frac{b}{a}} = -\frac{1}{\tan s},$$

$$\therefore \alpha_1 + \alpha_2 = 90° + s,$$

or, $$\alpha_1 - s = 90° - \alpha_2.$$

Therefore the angle of projection $\alpha_1 - s$ is equal to BOY. If, then, the angle of projection be kept constant, and the line of sight rotated about the point O, the locus of the extremity of the ranges measured on the line of sight will be the parabola OCT. Since

$$\alpha_2 = 90° - (\alpha_1 - s),$$

the equation to this curve is

$$y = x \cdot \frac{1}{\tan(a_1 - s)} - \frac{x^2}{4h}\left[1 + \frac{1}{\tan^2(a_1 - s)}\right].$$

It may be called the line of equal sight bar height. Its inclination to the vertical at the point D is $(a_1 - s)$.

In the theorem just proved, it is to be noticed that the angle a is not the angle of projection; and in order to investigate generally the subject of fire at objects which are not in the same horizontal plane with the gun, we shall resume equations (6) and (9); calling r the range on any inclined line: these are

$$r = \frac{2V^2}{g} \cdot \frac{\sin a \cdot \cos(a + s)}{\cos^2 s},$$

and $$R = \frac{2V^2}{g} \cdot \sin a \cdot \cos a.$$

From these, in which a is the angle of projection as it is always measured in practice, we have,

$$\frac{r}{R} = \frac{\cos(a + s)}{\cos a \cdot \cos^2 s}. \qquad (14)$$

By comparing (1) with the form to which it reduces when $s = 0$, it is apparent that if, with a constant value of a, s increases slightly from 0, the range will diminish (since there will be a small retardation along X); as, however, while $g \sin s$ increases, $g \cos s$ diminishes, the range may, and does, subsequently increase and pass through the value it had when $s = 0$. For negative values of s, (1) shows, when properly changed, that we shall always shoot over. In (14), let $a = 1°$ and $s =$ successively $1°, 2°, 2\frac{1}{2}°$; we find

$$\frac{r}{R} = .99999,\ 1.0000,\ 1.0001.$$

From (14),

$$\frac{r}{R} = \sec s\,(1 - \tan a \cdot \tan s).$$

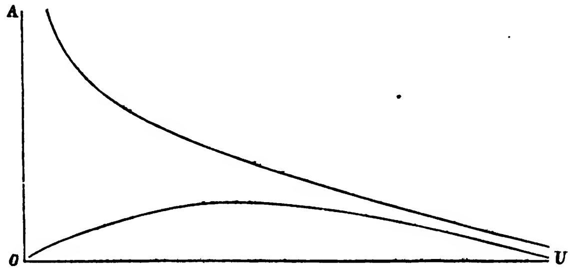
Put $\frac{r}{R} = 1$, $\sec s = u$, $\tan a = a$; then

$$1 = (1 + u^2)^{\frac{1}{2}}(1 - au). \qquad (15)$$

This is an equation of condition, which must be satisfied in order that $r = R$.

To trace its locus, square, by which we introduce the roots of $r = -R$; then we have

$$a^2u^3 - 2au^2 + a^2u + u - 2a = 0. \tag{16}$$



The locus is of the form shown: it passes through the origin at an inclination $\tan^{-1}\frac{1}{2}$, and the maximum point is given by $u = \pm 1.27202$ and $a = \pm .30028$. The similar branches in the 3d quadrant are not shown.

If now we assign any value to u ($\tan s$), it is evident that there is one value of a for which $r = R$. The upper branch of the curve will be found to correspond with those values for which $r = -R$. It is also apparent from the inclination of the curve at the origin, that, for very small angles, $r = R$, when $2a = u$, or $2a = s$. Conversely, for any assigned value of a ($\tan \alpha$), less than .300282 (or $\alpha = 16°42'50''$), there are two inclinations of the plane for which $r = R$.

For example, if $a = .10$, whence $\alpha = 5.7°$ about;

$u = .216$,	8.880,	10.911 about,	
whence $\beta = 12.2°$,	83.5°,	84.8°	"

By adding the last to 5.7°, we exceed 90°; this root, therefore, gives $r = -R$.

From (14) it follows that, for small angles, the error resulting from using on an incline a sight bar which has been marked for use on a horizontal plane is not great. This is also apparent from the small angle between the line of equal sight bar height at the point where it cuts the axis of X and the vertical. In practice, the assumption is frequently made that, for small angles, the range on any line is independent of its inclination to the horizontal. This is called the assumption of the Rigidity of the Trajectory, and consists, evidently, in assuming that the gun, trajectory, and line of sight—a chord—

may be rotated through a vertical angle without change of form. To insert this assumption analytically, we have merely to suppose, in equation (8), that the axes and trajectory may be turned through a small vertical angle without change of form. Of course this assumption violates the hypothesis upon which that equation rests. The assumption that the trajectory is rigid so nearly represents the truth when the angles are small, and is so convenient, that it is always adopted in practice.

The trajectory is assumed rigid in practice whenever a sight bar which has been marked for a gun at a certain height above the water is used when the height of the gun is changed, and also when the same sight bar is used to point at objects differing in vertical position. It is also assumed in plotting the trajectory by what is called the method of Polar Distortion. This will be described more at length hereafter. It is not often, in actual practice, that the height of the target subtends a greater angle than one degree at the gun, and within these limits the trajectory is sensibly rigid. Indeed, when we consider the attainable degree of accuracy in practical ballistic work generally, we may extend these limits to include $3°$ or $4°$. For example: suppose a 5-inch *ogival* projectile, weighing 64 lbs., is moving in air in a direction which makes an angle of $2°$ with the axis of X with a velocity of 1500 f.s., and that the axis of X is inclined upwards at $1°$, then the retardations along X due to gravity and the air are in the ratio $\frac{1}{239}$ nearly; those along Y are as $\frac{7}{1}$ nearly.

Examples.

1. A gun is pointed at an angle of projection of $1° \, 16' \, 31''$ above a line whose inclination is $45°$: the muzzle velocity being 1500 f. s., show that the time of flight is 2.93 s. and the range on the incline 1374 yards.

2. A shot being projected at the same velocity as in example 1 and at an angle of projection of $1° \, 16' \, 31''$ above a horizontal line; show that the time of flight is 0.207 s. and the range on this line 1036 yards.

3. Find the law connecting sight-bar height with range on the horizontal plane through the gun.

Solution. Putting, in (8), $y = 0$; we have $x = 0$, and $x =$ range $= x'$. Also, if the sight bar is at right angles with the axis of the

bore, and l is the distance between sights, $\tan\alpha = \frac{y'}{l}$, where y' is the height of the sight bar. We have, therefore, for the law required,

$$2V^2ly' - gx'y'^2 - gl^2x' = 0.$$

Prove, from this, that for small accidental changes of the angle of projection, the resulting errors in range are least when the angle of projection is 45°. (It is to be noted, however, that is not the case in practice. In the discussion above no account has been taken of accidental deviating forces; which, in high-angle fire, have a relatively long time to operate.)

4. Prove that the effect, on a vertical target, of accidental small variations of muzzle velocity are least when the velocity is greatest.

5. Prove that, in firing at a point whose coordinates with respect to the gun are (a, b), the point being higher than the gun; the least vertical error will result from a small inaccuracy in the estimation of distance if the trajectory have such a form that (a, b) shall be its vertex.

6. Devise a practical means of pointing a gun in case the point (a, b) in example 5 is to be the vertex.

7. In (8), putting $V = v$ and $\tan\alpha = u$, show that the simultaneous values of these variables for all trajectories through a point (a, b) are given by

$$v = \pm\left[\frac{ag}{2}\cdot\frac{1+u^2}{u-\frac{b}{a}}\right]^{\frac{1}{2}},$$

and thence show that the change in v necessary upon any change in u will be least when

$$u = \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}.$$

8. If R be the range on a horizontal line for the velocity V and angle of projection α, and r the range due to the same velocity and angle of projection on a line whose inclination is s; show that $R - r$ is a maximum when $\tan 2\alpha = \cos s . \tan\frac{s}{2}$; or, when the angles are very small, when $\alpha = \frac{s}{4}$.

9. Show, from the expression for $R - r$, derived as in example 8, that $R - r = 0$, when $\tan\alpha = \cos s . \tan\frac{s}{2}$.

10. Prove that the effect on a vertical target of small accidental variations of the angle of projection is least when $\alpha = 45° - \frac{s}{2}$; where α is the angle of projection and s the angle of sight. (The remark made in 3 applies in this case also.)

Numerous other examples, illustrating the subject of the motion of projectiles in a non-resisting medium, will be found in chapter XIII, *Text-Book of Ordnance and Gunnery.*

CHAPTER III.

THE RESISTANCE OF THE AIR.

The resistance of the air to projectiles moving in it with high velocities has long been known to be very considerable, and many attempts have been made to find a *law* which would account for the results obtained in practice.

Robins' experiments in 1742 are the first recorded attempts to obtain such a law, and were made by firing spherical bullets of lead from a small-arm and using a ballistic pendulum.

The following general statements were deduced:

(1) That for a velocity of the projectile not exceeding 1100 f. s., the resistance of the air varied as the *square of the velocity*.

(2) That at the velocity of 1100 f. s. (nearly the same as that with which sound is propagated through the air), this law of the resistance *changed*.

(3) That if the velocity exceeds 1100 f. s., then the absolute quantity of that resistance for the greater velocities was nearly three times as great as it should be by a comparison with the lower velocities.*

Hutton in 1790 made experiments with guns firing large spherical iron projectiles, and an improved ballistic pendulum, and from them deduced the law that if f denote the resistance of the air, and v the velocity of the projectile,

$$f \propto av + bv^2,$$

where a and b were constants determined by experiment.

General Didion, of the French army, in 1840 conducted a series of experiments at Metz, and proposed the following, which also consisted of two terms, the notation being the same as in the preceding,

$$f \propto .027\,(v^2 + .0023\,v^3).$$

This formula not having proved entirely satisfactory, a further series of experiments was made at Metz, in 1857, by Captain Welter (Professor in the School of Practical Artillery and Engineering), using an *electro-ballistic* pendulum. His conclusion was that the resistance

* Robins' "Tracts on Gunnery," by Hutton, p. 181.

of the air to *spherical* projectiles moving with high velocities was simply proportional to the *cube of the velocity;* or, $f \propto v^3$.

In 1860–1, at Gavre, experiments were made with *ogival-headed elongated* projectiles, by Prof. Hélie, of the French School of Naval Gunnery, from which he came to the conclusion that the resistance of the air was proportional to the cube of the velocity, at least as long as the axis of the projectile did not deviate much from the tangent to the trajectory described by the centre of mass.*

From 1865 to 1870, and again in 1878–80, series of most valuable experiments were carried on in England by Professor Bashforth, with spherical and ogival-headed projectiles, by means of his clock chronograph. The results showed that the resistance varies as follows:

For velocities between 900 and 1100 f. s. . . . $f \propto v^6$, approximately.
" " " 1100 " 1350 " . . . $f \propto v^3$ "
" " above 1350 f. s. . . . . $f \propto v^2$ "

It was also found that the resistance of the air *for the same velocity* and the same *form* of projectile, but of different calibers, varies exactly as the *square of the diameter* of the projectile. If the projectile presents a pointed head toward the opposing air, the resistance is less than if it is flat; and according to Bashforth's experiments we may say, that if the resistance to a hemispherical head be taken as unity, the value of the resistance for different shaped heads, the projectiles having the same diameter and weight and the same velocity, is as follows:

1. Hemispherical head, ...	1.
2. Hemispheroidal head, ...	0.7848
3. Ogival head, radius 1 caliber, . . .	0.8253
4. Ogival head, radius 2 calibers, . . .	0.7840
5. Flat head, ...	1.5122

Variation in the density of the air causes variation in the resistance, and it is assumed that it varies directly as the density.

Numerous formulas have been proposed to express the law of the resistance of the air, but it has been found to be very difficult to represent it as a simple continuous function which will admit of integration, hence it becomes convenient to assume that the resistance of

* "Traité de Balistique Experimentale," Hélie, 1865.

the air varies for all velocities as some power of the velocity, and to introduce a coefficient which is a *variable*, depending for its value upon the *form* of the projectile and upon the *velocity* with which it moves.

Professor Bashforth finds as an approximation that the resistance of the air varies as the cube of the velocity between certain limits, and the cubic law is that *assumed* to be true, but the exact amount of the resistance is found by the use of the variable coefficient before-mentioned, which is denoted by K. From what has before been stated we may conclude then that the resistance of the air may be expressed in mathematical symbols by

$$f = \frac{d^2}{g} K \left(\frac{v}{1000} \right)^3 \text{ in lbs. avoirdupois,} \qquad (1)$$

in which d is the diameter of the projectile in inches, and v the velocity in feet per second; the quantity $(1000)^3$ inserted in the denominator is for convenience only, since K is a very small decimal when it does not contain this factor; g appears in the denominator in order to reduce "absolute" to ordinary gravitation units.

The *effect* of the resistance of the air is to cause retardation in velocity, and the amount of retardation is known from the elementary law in dynamics,

$$-\frac{w}{g} r = f, \text{ whence } r = -\frac{gf}{w}, \qquad (2)$$

$\frac{w}{g}$ being the mass of the projectile. Combining (1) and (2) we have

$$r = -\frac{d^2}{w} K \left(\frac{v}{1000} \right)^3, \qquad (3)$$

which is the rate of the loss of velocity in feet per second caused by the resistance f in pounds.

From (1) it is seen that the *resistance* of the air is independent of the *weight* of the projectile; but the *retardation* is inversely as the weight; hence with projectiles of the same diameter but of different weights, the resistances are equal for the same initial velocity, but the retardation is less for the heavier one.

CHAPTER IV.

THE VARIABLE COEFFICIENT K.

Professor Bashforth's elaborate experiments before-mentioned furnish the best determined values of this quantity. The plan adopted was that of firing a projectile through a series of screens placed at a known distance apart, the first being placed at a suitable distance from the muzzle of the gun, from 50 to 100 feet usually, and recording the instant the projectile passed the screen, by means of the Bashforth Clock Chronograph and suitable electrical apparatus.

From the large number of experiments, the values of K found are the means of a large number of determinations for each velocity.

The method of the computation of the values of K from the observed times at each screen will now be given.

If we have any series of numbers increasing or decreasing in magnitude according to some law, and denoted by A, B, C, D, *etc.*, and if the 1st, 2d, 3d, etc., order of differences be taken, of which the first terms of each are a, b, c, d, etc. respectively, we may express each of the numbers of the series in terms of the first number A and the first terms of the successive orders of differences, and the coefficients of such an expression will follow the law of the Binominal Theorem. (See Ray's Algebra, par. 325.)

Thus,

$$B = A + a$$
$$C = A + 2a + b$$
$$D = A + 3a + 3b + c$$
$$E = A + 4a + 6b + 4c + d, \ etc., \ etc., \ etc.$$

Now suppose we replace A by t_s, the time taken to pass over the distance s, and which is some function $f(s)$ of s. Let B be replaced by $f(s + l)$, where l is a certain increment of distance, the interval between screens. Replace C by $f(s + 2l)$ when the distance is increased by another equal increment. Let the next terms be $f(s + 3l)$, $f(s + 4l)$, etc.

Let the 1st, 2d, 3d, etc. orders of differences be represented by Δ_1, Δ_2, Δ_3, etc.

Let the first differences

$f(s+l)-f(s)$ be represented by Δt_s,

$f(s+2l)-f(s+l)$ " Δt_{s+l}, etc., etc.

The second differences

$\Delta t_{s+l}-\Delta t_s$ be represented by $\Delta_2 t_s$,

$\Delta t_{s+2l}-\Delta t_{s+l}$ " $\Delta_2 t_{s+l}$, etc., etc.,

and the 3d, 4th, etc. differences in like manner. Adopting this notation we are naturally led to the following scheme:

	Δ_1	Δ_2	Δ_3	Δ_4	&c.
$f(s-4l)=t_{s-4l}$					
	Δt_{s-4l}				
$f(s-3l)=t_{s-3l}$		$\Delta_2 t_{s-4l}$			
	Δt_{s-3l}		$\Delta_3 t_{s-4l}$		
$f(s-2l)=t_{s-2l}$		$\Delta_2 t_{s-3l}$		$\Delta_4 t_{s-4l}$	
	Δt_{s-2l}		$\Delta_3 t_{s-3l}$		&c.
$f(s-l)=t_{s-l}$		$\Delta_2 t_{s-2l}$		$\Delta_4 t_{s-3l}$	
	Δt_{s-l}		$\Delta_3 t_{s-2l}$		&c.
$f(s)=t_s$		$\Delta_2 t_{s-l}$		$\Delta_4 t_{s-2l}$	
	Δt_s		$\Delta_3 t_{s-l}$		&c.
$f(s+l)=t_{s+l}$		$\Delta_2 t_s$		$\Delta_4 t_{s-l}$	
	Δt_{s+l}		$\Delta_3 t_s$		&c.
$f(s+2l)=t_{s+2l}$		$\Delta_2 t_{s+l}$		$\Delta_4 t_s$	
	Δt_{s+2l}		$\Delta_3 t_{s+l}$		
$f(s+3l)=t_{s+3l}$		$\Delta_2 t_{s+2l}$			
	Δt_{s+3l}				
$f(s+4l)=t_{s+4l}$					

Now from what precedes we have

$$
\begin{aligned}
f(s+nl) &= t_s + n\Delta t_s + \frac{n(n-1)}{\lfloor 2}\Delta_2 t_s + \frac{n(n-1)(n-2)}{\lfloor 3}\Delta_3 t_s + \text{\&c.} \\
&= t_s + n\{\Delta t_s - \tfrac{1}{2}\Delta_2 t_s + \tfrac{1}{3}\Delta_3 t_s - \tfrac{1}{4}\Delta_4 t_s + \tfrac{1}{5}\Delta_5 t_s - \text{\&c.}\} \\
&\quad + n^2\{\tfrac{1}{2}\Delta_2 t_s - \tfrac{1}{2}\Delta_3 t_s + \tfrac{11}{24}\Delta_4 t_s - \tfrac{10}{24}\Delta_5 t_s + \tfrac{137}{180}\Delta_6 t_s + \text{\&c.}\} \\
&\quad + \text{\&c. \&c.}
\end{aligned}
$$

Expanding t_{s+nl} by Taylor's theorem, we have

$$
t_{s+nl} = t_s + \frac{dt_s}{ds}\frac{nl}{1} + \frac{d^2t_s}{ds^2}\frac{n^2l^2}{\lfloor 2} + \frac{d^3t_s}{ds^3}\frac{n^3l^3}{\lfloor 3} + \text{\&c.}
$$

And equating the coefficients of the first and second powers of n in the two expansions of t_{s+nl}, we have

$$
l\frac{dt_s}{ds} = \Delta t_s - \tfrac{1}{2}\Delta_2 t_s + \tfrac{1}{3}\Delta_3 t_s - \tfrac{1}{4}\Delta_4 t_s + \tfrac{1}{5}\Delta_5 t_s - \text{\&c.} \qquad (1)
$$

$$
l^2\frac{d^2t_s}{ds^2} = \Delta_2 t_s - \Delta_3 t_s + \tfrac{11}{12}\Delta_4 t_s - \tfrac{10}{12}\Delta_5 t_s + \tfrac{137}{180}\Delta_6 t_s - \text{\&c.} \qquad (2)
$$

Now consulting the scheme, we see that $t_s = t_{s-1} + \Delta t_{s-1}$, so that we have

$$\begin{aligned} \Delta_2 t_s &= \Delta_2 t_{s-1} + \Delta_3 t_{s-1} \\ - \quad \Delta_3 t_s &= -\Delta_3 t_{s-1} - \Delta_4 t_{s-1} \\ + \tfrac{11}{12}\Delta_4 t_s &= +\tfrac{11}{12}\Delta_4 t_{s-1} + \tfrac{11}{12}\Delta_5 t_{s-1} \\ - \tfrac{10}{12}\Delta_5 t_s &= -\tfrac{10}{12}\Delta_5 t_{s-1} - \tfrac{10}{12}\Delta_6 t_{s-1} \\ + \tfrac{137}{180}\Delta_6 t_s &= +\tfrac{137}{180}\Delta_6 t_{s-1} + \tfrac{137}{180}\Delta_7 t_{s-1} \end{aligned}$$

Taking the sum of both sides of these equations, we have

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta_2 t_{s-1} - \tfrac{1}{12}\Delta_4 t_{s-1} - \tfrac{18}{180}\Delta_6 t_{s-1} + \&c., \tag{3}$$

and which may be still further reduced by putting for $-\tfrac{1}{12}\Delta_4 t_{s-1}$ and $-\tfrac{18}{180}\Delta_6 t_{s-1}$ their values in terms of lower orders of differences, so that finally we have

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta_2 t_{s-1} - \tfrac{1}{12}\Delta_4 t_{s-2} + \tfrac{1}{90}\Delta_6 t_{s-3} - \&c. \tag{4}$$

The fourth differences are so small that they may generally be neglected.

Now to find the velocity and retardation at each screen, we have, since $v = \frac{ds}{dt_s}$ and $r = \frac{d^2 s}{dt_s^2}$, but $r = \frac{d}{dt_s}\left(\frac{ds}{dt_s}\right)$, and making s the independent variable,

$$r = -\frac{ds}{dt_s}\cdot\frac{d^2 t}{dt_s^2} = -\frac{ds}{dt_s}\cdot\frac{d^2 t_s}{dt_s^2}\cdot\frac{ds^2}{ds^2} = \frac{d^2 t_s}{ds^2}\cdot\left(\frac{ds}{dt_s}\right)^3 = -v^3\cdot\frac{d^2 t_s}{ds^2} \tag{5}$$

From equation (1) we have

$$v = \frac{ds}{dt_s} = \frac{l}{\Delta t_s - \frac{1}{2}\Delta_2 t_s + \frac{1}{8}\Delta_3 t_s - \&c.} \tag{6}$$

Resuming equation (5), substituting the value of $\frac{d^2 t_s}{ds^2}$ from (4), we have

$$r = -\frac{v^3}{l^2}\left(\Delta_2 t_{s-1} - \tfrac{1}{12}\Delta_4 t_{s-2} + \tfrac{1}{90}\Delta_6 t_{s-3} - \&c.\right)$$

From the preceding chapter we have

$$r = -K\frac{d_2}{w}\left(\frac{v}{1000}\right)^3,$$

$$\therefore\ K = \frac{w}{d^2}\,\frac{(1000)^3}{l^2}\left(\Delta_2 t_{s-1} - \tfrac{1}{12}\Delta_4 t_{s-2} + \&c.\right) \tag{7}$$

Applying the formulas to an example, suppose a hollow elongated shot weighing 23.84 lbs., its diameter being 4.92 inches, to be fired

through 10 screens, and the instant of the shot passing each screen to be recorded by a suitable instrument, as follows:

Screen.	Passed at seconds.	Δ_1	Δ_2
1	3.0526		
		+ 1090	
2	3.1616		+ 24
		+ 1114	
3	3.2730		+ 24
		+ 1138	
4	3.3868		+ 25
		+ 1163	
5	3.5031		+ 25
		+ 1188	
6	3.6219		+ 25
		+ 1213	
7	3.7432		+ 24
		+ 1337	
8	3.8669		+ 24
		+ 1261	
9	3.9930		+ 25
		+ 1286	
10	4.1216		

The screens are 150 feet apart $= l$. The velocity v_5 at the 5th screen is then

$$v_5 = \frac{150}{\Delta t_5 - \frac{1}{2}\Delta_2 t_5} = \frac{150}{.1188 - \frac{1}{2}.0025} = \frac{300}{.2351} = 1276.1 \text{ f. s.}$$

and the value of K is

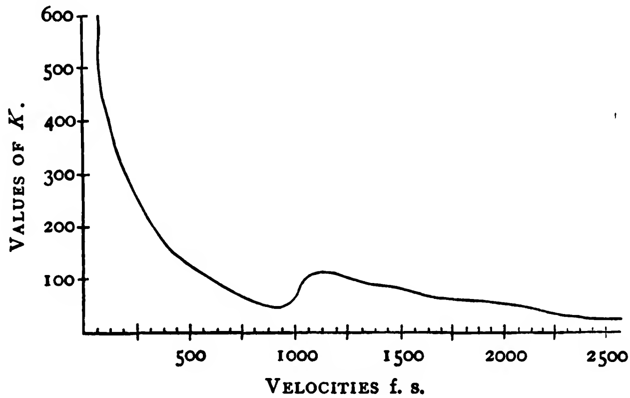
$$K_v = + \frac{23.84\,(1000)^3 \times .0025}{(4.92)^2(150)^2} = 109.4.$$

A slight correction is sometimes necessary on account of the density of the air at the time of the experiment varying a little from the standard.

In Table I at the end of the book are given the values of K for ogival-headed projectiles, radius of head $1\frac{1}{2}$ calibres, for values of the velocity from 100 f. s. to 2800 f. s. for every 10 feet. A cubic foot of air at a temperature of 62° F. under a barometric pressure of 30 inches of mercury is assumed to weigh 534.22 grains, and the values of K correspond to these standard conditions. These values are

used in the computation of the tables which follow, and if they are used under conditions of the atmosphere which differ from the standard, a small correction is necessary to their correct use, as will be hereafter shown.

The annexed curve shows the variations of K graphically, the abscissae representing velocities, and the ordinates the corresponding values of K.



It will be noticed that a marked maximum summit occurs at velocities near or just exceeding that of the velocity of sound. The reason for this variation in regularity is not known, but the fact that it exists adds to the difficulty of the discovery of some general law for the resistance of the air.

CHAPTER V.

THE MOTION OF PROJECTILES IN AIR. SLADEN'S METHOD OF SOLVING CERTAIN BALLISTIC PROBLEMS.

§I.

If r denote the retardation of a projectile moving in air in standard conditions, then, since in Bashforth's equation

$$r = -\frac{d^2}{w} K \left(\frac{v}{1000}\right)^3$$

the values of K for all ordinary velocities have been determined by experiment and tabulated (see Table I), we may find the retardation, and, consequently, the force resisting any projectile. Also, since

$$r = \frac{dv}{dt} = \frac{vdv}{ds},$$

where ds and dt are respectively the differentials of distance passed over and of time, we have

$$\frac{dv}{dt} = \frac{vdv}{ds} = -\frac{d^2}{w} . K \left(\frac{v}{1000}\right)^3 \tag{1}$$

Equations (1) are of great importance; and, since they give a relation between time and velocity and one between distance passed over and velocity, they would furnish the complete solution of ballistic problems, in which we ordinarily require relations between time of flight, range, and velocity; if gravity did not act. In (1), however, K is not constant; and therefore integration is impossible, unless the form of the relation $K = f$ (velocity) be known. The form of this relation to any degree of approximation might evidently, from what has gone before, be found; but when the complexity of this relation and the consequent difficulty of integration is considered, it will be apparent that it is simpler in practice to integrate the equations

$$\frac{dv}{dt} = -\frac{d^2}{w} . K \left(\frac{v}{1000}\right)^3, \tag{2}$$

$$\frac{vdv}{ds} = -\frac{d^2}{w} K \left(\frac{v}{1000}\right)^3, \tag{3}$$

over such narrow limits of change of velocity that a mean value of K between these limits may be taken from the tabular values of this latter quantity. By proceeding in this way by successive steps we may form two tables, connecting time and velocity, and distance and velocity, which will serve for the solution of problems, by picking out the tabular values required.

Let V_0 be the velocity of a projectile moving in air at the origin of time, V_1 its velocity at a subsequent time, and let T be the whole time elapsed while the velocity changes from V_0 to V_1; then (2) gives, if K be the mean value of that quantity between the limits V_0 and V_1,

$$\frac{d^2}{w}T = \frac{(1000)^3}{2K}\left[\frac{1}{V_1^2} - \frac{1}{V_0^2}\right] \tag{4}$$

Also, if S be the whole distance passed over by the projectile while the velocity changes from V_0 to V_1, (3) gives

$$\frac{d^2}{w}S = \frac{(1000)^3}{K}\left[\frac{1}{V_1} - \frac{1}{V_0}\right] \tag{5}$$

It is evident, however, that we cannot determine the proper values of K to use in (4) and (5), unless V_0 and V_1 are close together. If now from (4) we write the series of equations

$$\frac{d^2}{w}T_1 = \frac{500}{K_1}\left[\left(\frac{1000}{V_2}\right)^2 - \left(\frac{1000}{V_1}\right)^2\right],$$

$$\frac{d^2}{w}T_2 = \frac{500}{K_2}\left[\left(\frac{1000}{V_3}\right)^2 - \left(\frac{1000}{V_2}\right)^2\right],$$

$$\frac{d^2}{w}T_3 = \frac{500}{K_3}\left[\left(\frac{1000}{V_4}\right)^2 - \left(\frac{1000}{V_3}\right)^2\right],$$

. . . .

it is clear that we may, by taking the two values of the velocity in each equation more and more near to each other, form a table connecting time and velocity (such as Table III) to any required degree of accuracy. Similarly, we may write from (5),

$$\frac{d^2}{w}S_1 = \frac{(1000)^2}{K_1}\left[\frac{1000}{V_2} - \frac{1000}{V_1}\right],$$

$$\frac{d^2}{w}S_2 = \frac{1000}{K_2}\left[\frac{1000}{V_3} - \frac{1000}{V_2}\right],$$

$$\frac{d^2}{w}S_3 = \frac{1000}{K_3}\left[\frac{1000}{V_4} - \frac{1000}{V_3}\right],$$

. . . .

Since the value of the factor $\frac{d^2}{w}$ is different in different guns, it will be more convenient to make out the tables for a projectile in which $\frac{d^2}{w} = 1$; and the tables are so made out. Also in *all* the tables given, d, the diameter of the projectile, must be taken in inches; w, the weight of the projectile, in pounds; the velocities in feet per second; the distances passed over in feet; and the times in seconds.

Examples.

1. It is required to find in how long a time a 6-inch projectile, whose weight is 100 lbs., and which is moving with a velocity of 2800 f. s., will have its velocity reduced to 2700 f. s.

Table I shows that the value of K is sensibly 52 between these limits, and (4) gives

$$T = \frac{100}{36} \cdot \frac{500}{52} \cdot \left[\left(\frac{1000}{2700}\right)^2 - \left(\frac{1000}{2800}\right)^2\right]$$

$$\therefore\ T = 0.2572 \text{ second.}$$

To solve this problem directly by Table III, we would proceed as follows: Entering the table with the velocity as argument, we find the time a projectile in which $\frac{d^2}{w} = 1$ would lose (or gain) the said velocity from an arbitrary origin. For example, since the initial value of the time in Table III is 75.399, and the initial value of the velocity 100 f. s., we find that a projectile in which $\frac{d^2}{w} = 1$ would have its velocity reduced (or increased, if the same law of change of velocity operated in the inverse sense) from 2800 f. s. to 100 f. s. in $234.5001 - 75.399 = 159.1011$ secs. Thus with the 6-inch projectile above defined, we find in the table opposite 2800 and 2700 the numbers 234.5001 and 234.4075; thus this change of velocity would be caused in

$$T = \frac{100}{36}\,(234.5001 - 234.4075),$$

$$\therefore\ T = .2572 \text{ second.}$$

Note.—It is well to observe that as a matter of convenience the tables have been made out as though the projectile was gaining, instead of losing, velocity (this is the case in Table IV also). An

inspection of the tables shows that the time and velocity increase together.

2. For what distance must the 6-inch projectile whose weight is 100 lbs. move *in air* in order to have its velocity reduced from 2800 f. s. to 2700 f. s.? In Table II, entering with 2800 f. s. and 2700 f. s., we find the tabular values 47574.6 feet and 47320.2 feet; therefore, for the distance required,

$$S = \frac{100}{36}(47574.6 - 47320.2),$$

$$\therefore S = 706.7 \text{ feet.}$$

3. Find the resistance of the air and the retardation of a projectile in which $d = 12.5$ inches and $w = 802.25$ lbs., when moving in air with a velocity of 1400 f. s.

Resistance = 1394.5 lbs.
Retardation = 55.96 f. s.

4. Find the time in which the projectile in example 3 will have its velocity reduced to 1344.04 f. s.

$T = 1.0493$ secs.

5. To find the muzzle velocity of the 3-inch B. L. R., in which $d = 3$, $w = 7$, it is fired through two screens, and its velocity at the middle point between the screens is found to be 1062 f. s. This point being situated 70 feet from the muzzle of the gun, find the muzzle velocity.

Problems of this nature lead to a convenient manner of writing (4) and (5) in the use of the tables. These equations may evidently be written

$$\frac{d^2}{w} T = T_{V_1} - T_{V_2} \tag{6}$$

$$\frac{d^2}{w} S = S_{V_1} - S_{V_2}; \tag{7}$$

where T_V and S_V signify the tabular numbers in Tables III and II corresponding to the velocity V. Thus we may solve the problem just stated thus:

$$\frac{9}{7} 70 = S_{V_1} - S_{1062}$$

$$\therefore S_{V_1} = 40577.6 + 90 = 40667.6$$

$$\therefore V_1 = 1071.9;$$

the muzzle velocity required.

6. To find the time of flight with a projectile in which $w = 64$, $d = 6.3$, when projected with a volocity of 1457 f. s., for a range of 1000 *yards*. For this velocity and range the length of the curved path which the shot will describe may for practical purposes be taken equal to the length of its chord, which is the range stated. In a case of this nature we must first find the velocity at the end of the range by (7), and then (6) will evidently furnish the solution.

$T = 2.3575$.

7. Prove that the rate of loss of velocity of ogival projectiles is inversely proportional to their caliber if their form is *similar;* that is, if their heads are so formed that their weights are proportional to the cube of the caliber.

8. A projectile in which $d = 6$ inches, $w = 72$ lbs., is fired at an iron target 100 yards from it with a muzzle velocity of 2000 f. s. Find the striking velocity and time of flight. Here the arc considered of the trajectory is sensibly equal to its chord, the range.

Striking velocity $= 1959.3$ f. s.
Time of flight $= 0.1516$ sec.

9. The same projectile as in 8 is fired at an iron target 50 yards from it, and must have 2000 f. s. velocity to penetrate. Find muzzle velocity and time of flight.

$V = 2020.8$
$T = .0746$

10. The same projectile is fired from a gun which is 55 feet from a target screen, a second screen being 90 feet from the first. By means of a chronograph, the interval of time between the rupturing of the two screens is found to be 0.05 sec. Find the muzzle velocity. The velocity at the middle point between the two screens is, very approximately, $V_2 = \frac{90}{.05} = 1800$ f. s.

Muzzle velocity $= 1812.6$

11. Find the resistance of the air to a base-ball of 3 inches diameter when moving at a velocity of 100 f. s., assuming the resistance to the sphere and standard ogival to be in the ratio 1 : .8.

Resistance $= .202$ lb.

12. An ogival projectile, in which $d = 6.3$ inches and $w = 64$ lbs., is dropped point foremost in the air. Find the greatest velocity it will acquire. Its acceleration will be zero when the resistance of the air is equal to the projectile's weight; consequently we find, by continual approximation to the value of K,

Velocity required $= 884$ f. s., about.

§ II.

The above methods furnish the solution of some of the simpler ballistic problems which occur; and it is evident that whenever the distance passed over enters, their application must be confined to those cases in which the range, a chord, is sensibly equal for practical purposes to the curved path the shot actually pursues. It is difficult to assign precise limits beyond which their use should not extend, as these limits depend upon the ranges considered, the velocity, the caliber, as well as upon other things of minor importance. For medium guns with high velocity, the limit should, perhaps, not exceed an angle of departure of 3°, which would correspond to a range of about 2500 yards.

Within these limits, (2) and (3) furnish the solution of all problems in so far as the time and one coordinate, the horizontal one, along which the ranges are measured, is concerned. If now, since the angles of departure are small, and consequently the vertical component of the velocity inconsiderable, we assume that the vertical motion of the shot is affected by gravity only; we have, taking the positive direction of the axis of Y upwards through the muzzle of the gun,

$$\frac{d^2y}{dt^2}=-g. \tag{8}$$

Integrating, we have

$$\frac{dy}{dt}=-gt+C.$$

Here, since when $t=\frac{T}{2}$, T being the whole time of flight over any arc of the trajectory whose extremities are in the same horizontal plane, $\frac{dy}{dt}=0$, $C=g\frac{T}{2}$. Integrating again, we have

$$y=\frac{gt}{2}(T-t). \tag{9}$$

The second constant of integration being zero because y and t are zero together.

The use of (9), together with (2) and (3), or the substitution of Tables II and III for the latter, constitute what is known as Sladen's Method; having been first proposed by Lieutenant-Colonel Sladen, of the British Army. The limits within which this method will give correct results are the same as those within which the range may be considered equal in length to the trajectory; since, within these limits, the vertical component of the velocity will always be small.

In this method it will be observed that the angle does not enter directly. In any case, however, in which we know the vertical and horizontal velocities approximately, we may, from the triangle of velocities, find the angle approximately.

Examples.

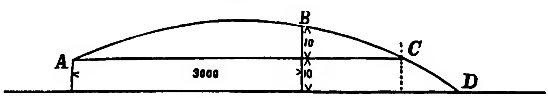
1. Explain why, for any value of y in (9), there are two unequal values of t. This will be evident by drawing the approximate form of the trajectory.

2. It is required to plot the trajectory of a small-arm bullet in which $d = .45$ inch, $w = 480$ grains $= .0686$ lb., and muzzle velocity $= 1353$ f. s., for a range of 500 yards. We must first find the time of flight for 500 yards, as this is T in (9). This is found to be 1.4355 seconds. Then, if it be determined to get a point at each 100 yards, we find the time of flight over 100 yards. This is $= .2362$ second. Then (9) gives

$$y_{100} = 4.56 \text{ feet.}$$

This is the height of the projectile at the end of the first 100 yards. Similarly at 200, 300, 400, we find the y to be 7.53, 8.20, 5.89 feet respectively.

3. An 8-inch rifle, in which $d = 8$ inches, $w = 180$ lbs., and muzzle velocity $= 1450$ f. s., is fired at the middle point of the height of a target which is 20 feet high and is really at 1000 yards from the gun. The gun being mounted 10 feet above the water, find what overestimate of the distance may be made without missing the target.



$$\tfrac{64}{180}\,3000 = S_{V_A} - S_{V_B} \qquad \therefore\; V_B = 1144.6$$

$$\tfrac{64}{180}\,T_{AB} = T_{V_A} - T_{V_B} \qquad \therefore\; T_{AB} = 2.2385$$

$$y = \frac{gt}{2}\,(T - t) \qquad \therefore\; T = \frac{2y + gt^2}{gt}$$

In this last equation, put $t = T_{AB}$, and $y = 10$, then $T = T_{AC}$, hence

$$T_{AC} = 2.5161$$

$$\tfrac{64}{180}\,T_{AC} = T_{V_A} - T_{V_C}, \qquad \therefore\; V_C = 1224.3$$

$$\tfrac{64}{180}\,S_{BC} = S_{V_B} - S_{V_C}, \qquad \therefore\; S_{BC} = 342.6 \text{ feet;}$$

the overestimate required.

4. A Hotchkiss revolver-cannon, in which $d = 1.456$ inches, $w =$ 1 lb., and muzzle velocity $= 1317$ f. s., is fired at a point at its own height in a target which is 800 yards distant. If the target is 4 feet higher than the point aimed at, find what overestimate of the distance may be made without missing.

Overestimate $=$ 88 feet, nearly.

5. Find the underestimate which may be made without missing in the case of Example 3. It will be necessary here to compute backwards, beginning at the shortest point in the water which may be struck, which is the foot of the target.

Underestimate $=$ 347.9 feet.

6. A projectile is fired from a 60-pdr. B. L. R. under the following conditions: $d = 5.3$ inches, $w = 46$ lbs., muzzle velocity $= 1071$ f. s., angle of departure $= 2°$, range $= 760$ yards; it is required to find the angle of fall.

This example is intended to illustrate an important application of Table IV; from which, by entering successively with two velocities as argument, we are enabled to find the whole angle through which the direction of motion of the shot has rotated while the velocity changed from its first to its second value. The manner of computing this table will be given hereafter. In the equation written at the top of the table, D is a change of angle in degrees, and D_V are tabular values corresponding to a velocity V.

By the space and velocity table (Table II), we find the striking velocity to be 958.6 f. s. Then, substituting values in the equation

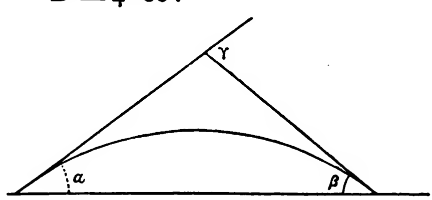
$$\frac{d^2}{w} D = D_{V_1} - D_{V_2},$$

we have

$$\frac{\overline{5 \cdot 3}^2}{46} D = D_{1071} - D_{958.6}$$

$$D = \frac{46}{\overline{5 \cdot 3}^2} [80.9606 - 78.4391)$$

$$D = 4° \; 08'.$$



This is the angle γ in the figure. Consequently, the angle of fall is $2° \; 08'$.

§ III.

To determine the correction to be applied in the computation of trajectories for variations of thermometer and barometer.

All the tabular values have been reduced to a standard atmospheric condition of 62° F. and 30 inches of mercury, in which a cubic foot of air weighs 534.22 grains. If the atmospheric conditions vary from this standard, the results derived from the tables may be approximately corrected as follows: The general formula for the use of Table II may be written

$$\frac{d^2}{w}\left(1+\frac{\Delta}{534.22}\right)S = S_{V_1} - S_{V_2};$$

and similarly for the other tables. To find Δ, let W_0 be the weight of air as defined; and W its weight in any conditions of thermometer and barometer, t and β. We have

$$pv = K(1+at);$$

also $\frac{W}{W_0} = \frac{v_0}{v}$. If the pressure only varies, $\frac{W}{W_0} = \frac{p}{p_0}$; and if the temperature only varies, $\frac{W}{W_0} = \frac{v_0}{v} = \frac{1+at_0}{1+at}$. If they both vary,

$$\frac{W}{W_0} = \frac{1+at_0}{1+at} \cdot \frac{p}{p_0}.$$

We have also $\frac{p}{p_0} = \frac{\beta}{\beta_0}$, $W_0 = 534.22$, $\beta_0 = 30$, $a = .002178$. If we reckon t from t_0, the latter is zero. Hence

$$W = \frac{534.22}{1+.002178 \times t} \cdot \frac{\beta}{30};$$

and

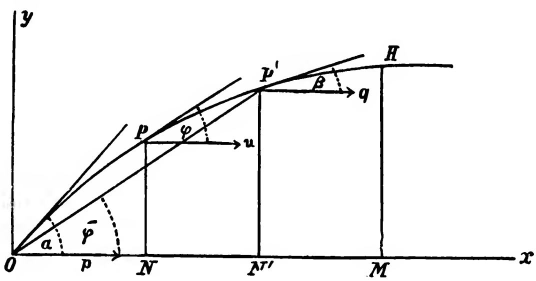
$$\Delta = W - W_0.$$

It is evident that this correction should be applied in computing the higher parts of very high-angled trajectories.

CHAPTER VI.

TRAJECTORIES IN THE AIR. NIVEN'S METHOD.

In the calculations of the range, time of flight, and other elements of the trajectory of a projectile, the deviation of the axis of the projectile from the trajectory, and the drift, are neglected for the sake of simplicity. The effect of these deviations on the quantities to be determined is small, especially at the lower angles of departure.



In the figure let

$v =$ the velocity at any point of the trajectory.
$u =$ the horizontal component of v.
$\varphi =$ the inclination of the direction of motion to the horizontal line in the plane of the trajectory.
$t =$ the time.
$x =$ the horizontal distance from the point of departure.
$y =$ the vertical distance.

Suppose O the point of departure, a the angle of departure, OH the ascending path of the projectile, H the vertex of the trajectory, Ox the horizontal line in the plane of the trajectory, Oy the vertical line, P the position of the projectile at any time.

Considering now the motion of the projectile over a portion of the arc OH, as OP', a and β being the inclination of the direction of motion to the horizontal line Ox, at the beginning and end of the arc, and p and q the horizontal velocities, that is to say, the values of u at those points. Resuming the value of the retardation from Chapter III.

The retardation in the direction of the horizontal line Ox is

$$\frac{d^2x}{dt^2} = \frac{du}{dt} = - r \cos \varphi, \tag{1}$$

in which $r = K \frac{d^2}{w} \left(\frac{v}{1000}\right)^3$.

Now if ρ be the radius of curvature of any small part ds of the trajectory inclined at an angle φ to the horizontal, then ds subtends an angle $d\varphi$ at the centre of curvature; the acceleration in the direction of the normal is $\frac{v^2}{\rho}$ and this must equal the normal component of the acceleration of gravity,

or
$$\frac{v^2}{\rho} = g \cos \varphi. \tag{2}$$

Now
$$\rho = - \frac{ds}{d\varphi},$$

the negative sign being taken, since φ diminishes as s increases. But

$$\frac{ds}{dt} = v$$

$$\therefore \rho = - v \frac{dt}{d\varphi}. \tag{3}$$

Whence eliminating ρ between (2) and (3),

$$v \frac{d\varphi}{dt} = - g \cos \varphi. \tag{4}$$

$$\therefore \frac{d\varphi}{du} = \frac{g}{rv},$$

or
$$d\varphi = g \frac{du}{vr}.$$

Integrating, the limits for φ being α and β, and for u, p and q,

$$\int_\beta^\alpha d\varphi = g \int_q^p \frac{du}{vr},$$

or
$$\alpha - \beta = g \int_q^p \frac{du}{vr}. \tag{5}$$

From equation (1) we have

$$dt = - \frac{du}{r \cos \varphi};$$

hence the time while α changes to β is

$$T=\int_{q}^{p}\frac{du}{r\cos\varphi}. \tag{6}$$

Since $\dfrac{dx}{dt}=u$,

$$X=\int_{q}^{p}udt=\int_{q}^{p}\frac{udu}{r\cos\varphi}, \tag{7}$$

and since $\dfrac{dy}{dx}=\tan\varphi$,

$$Y=\int_{q}^{p}\frac{u\tan\varphi\,du}{r\cos\varphi}. \tag{8}$$

Equations 5, 6, 7 and 8 are the general forms of the integrals which give the characteristic quantities in the motion of a projectile.

In order to solve them rigidly, the law of the resistance of the air must be known. Since no simple law properly expresses the relation between the quantity r and the velocity, let us assume that r is some function $f(v)$ of v, and taking the X integral as an example, see how it may approximately be calculated.

$$X=\int_{q}^{p}\frac{udu}{r\cos\varphi},$$

or

$$X=\int_{q}^{p}\frac{udu}{f(v)\cos\varphi},$$

$v=u\sec\varphi$; hence

$$X=\int_{q}^{p}\frac{udu}{f(u\sec\varphi)\cos\varphi}.$$

Now it is evident that there must be some mean value of φ such that

$$X=\int_{q}^{p}\frac{udu}{f(u\sec\bar{\varphi})\cos\bar{\varphi}},$$

if we denote that value of φ by $\bar{\varphi}$.

Referring to the figure, if the arc is short, the angle $\bar{\varphi}$ is approximately that made by the chord OP' with the horizontal. We shall see later, however, how the value of $\bar{\varphi}$ is determined.

Suppose now that $u=z\cos\bar{\varphi}$, then $z=u\sec\bar{\varphi}$, and $du=\cos\bar{\varphi}\,dz$, so that

$$X=\int_{q\sec\bar{\phi}}^{p\sec\bar{\phi}}\frac{z\cos\bar{\varphi}dz}{f(z)}.$$

Now, the value of $f(z)$, as determined by Professor Bashforth, is

$$r=f(z)=K\frac{d^2}{w}\left(\frac{z}{1000}\right)^3,$$

so that,

$$\frac{d^2}{w} X = \cos \bar{\varphi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dz}{Kz^3}. \tag{9}$$

Similarly

$$\frac{d^2}{w} Y = \sin \bar{\varphi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dz}{Kz^3}. \tag{10}$$

It will be seen that the values of X and Y both depend on the value of the integral,

$$\int_z^z \frac{(1000)^3 dz}{Kz^3} = S_z - S_z,$$

but the values of this integral have been computed and tabulated, see Table II, so that equations (9) and (10) may be written

$$\frac{d^2}{w} X = \cos \bar{\varphi} \, (S_{p \sec \bar{\phi}} - S_{q \sec \bar{\phi}}) \tag{12}$$

$$\frac{d^2}{w} Y = \sin \bar{\varphi} \, (S_{p \sec \bar{\phi}} - S_{q \sec \bar{\phi}}). \tag{13}$$

This mode of treatment may be employed in the reduction of the T integral, and equation (7) reduces to

$$\frac{d^2}{w} T = \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dz}{Kz^3};$$

or

$$\frac{d^2}{w} T = T_{p \sec \bar{\phi}} - T_{q \sec \bar{\phi}}, \tag{14}$$

since the value of the integral is given in Table III.

It will be observed that the determination of the distance and time integrals both depend upon the value of the quantity $q \sec \bar{\varphi}$, in which q is an unknown quantity; but this can be determined by the consideration of equation (5),

$$\alpha - \beta = g \int_q^p \frac{du}{vr} = g \int_q^p \frac{du}{vf(v)},$$

or

$$\alpha - \beta = g \int_q^p \frac{du}{u \sec \varphi f(u \sec \varphi)}.$$

As before, suppose φ has a mean value $\bar{\varphi}$, and suppose $\alpha - \beta$ expressed in degrees $= D$: then

$$D = \frac{180}{\pi} g \int_q^p \frac{du}{u \sec \bar{\varphi} \, f(u \sec \bar{\varphi})}.$$

Put $u = z \cos \bar{\varphi}$ as before, then

$$D \sec \bar{\varphi} = \frac{180 g}{\pi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{dz}{vf(z)},$$

but, as before,
$$f(z)=\frac{d^2}{w}K\left(\frac{z}{1000}\right)^3,$$

so that
$$\frac{d^2}{w}D=\cos\bar{\varphi}\,\frac{180g}{\pi}\int_{q\sec\bar{\phi}}^{p\sec\bar{\phi}}\frac{(1000)^3\,dz}{Kz^4}. \tag{15}$$

The value of the integral
$$\frac{180g\,(1000)^3}{\pi}\int\frac{dv}{Kv^4},$$

has been computed by Mr. Niven, using Bashforth's values of K, and these values are tabulated in Table IV.

This table, connecting degrees and velocity, is denoted by D_v, and evidently if this integral is represented by D_v, equation (15) becomes
$$\frac{d^2}{w}D=\cos\bar{\varphi}\,(D_{p\sec\bar{\phi}}-D_{q\sec\bar{\phi}}), \tag{16}$$

from which $q\sec\bar{\varphi}$ can be determined, and hence the value of q be found.

The values of $\bar{\varphi}$ to be used for the X and Y integrals are
$$\bar{\varphi}=\frac{a+\beta}{2}+\frac{1}{3}\frac{p-q}{p+q}(a-\beta)$$

for the ascending branch, and
$$\bar{\varphi}=\frac{a+\beta}{2}-\frac{1}{3}\frac{p-q}{p+q}(\beta-a)$$

for the descending branch of the trajectory.

The mean angle $\bar{\varphi}$ for the time integral is not quite the same, being
$$\bar{\varphi}'=\frac{a+\beta}{2}+\frac{1}{6}\frac{p-q}{p+q}(a-\beta)$$

for the ascending branch, and
$$\bar{\varphi}'=\frac{a+\beta}{2}-\frac{1}{6}\frac{p-q}{p+q}(\beta-a)$$

for the descending branch. The method of deducing these values will be given later on.

There is no means of determining the exact value of the mean angle $\bar{\varphi}$ in the case of the D integral. Since, however, $\sec\bar{\varphi}$ varies very slowly for the greater part of the trajectory, it will suffice in this equation to put $\bar{\varphi}=\frac{a+\beta}{2}$ for low angles of departure. For high

angles of departure, $\bar{\varphi}$ will be more accurately determined from the formula

$$\tan \bar{\varphi} = \frac{\tan \alpha + \tan \beta}{2}.$$

This may be shown in the following manner:

Equation (15), in which K is assumed to be constant, may be put in the form

$$D = C \cos \bar{\varphi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{dz}{z^4},$$

in which C represents the product of all the constants. Effecting the integration, we have

$$D = \frac{C \cos \bar{\varphi}}{3} \left(\frac{1}{q^3} - \frac{1}{p^3} \right) \cos^3 \bar{\varphi}$$

$$\log D = \log \left[\frac{C}{3} \left(\frac{1}{q^3} - \frac{1}{p^3} \right) \right] + 4 \log \cos \bar{\varphi},$$

whence, differentiating with respect to D and $\bar{\varphi}$,

$$\frac{dD}{Dd\bar{\varphi}} = -4 \tan \bar{\varphi}.$$

Whence it is evident that the rate of change of D as a function of $\bar{\varphi}$ compared with D, is proportional to $\tan \bar{\varphi}$; hence it is more accurate to use a mean value of $\tan \bar{\varphi}$ than a mean value of $\bar{\varphi}$.

The determination of $\bar{\varphi}$.

The following solutions are given by Professor J. M. Rice, U. S. Navy:

Equation (1) may be written

$$\frac{du}{dt} = -cv^3 \cos \varphi, \tag{17}$$

in which

$$c = K \frac{d^2}{w(1000)^3},$$

and equation (4)

$$v \frac{d\varphi}{dt} = -g \cos \varphi, \tag{18}$$

and dividing (17) by (18) we have

$$\frac{du}{d\varphi} = \frac{cv^4}{g},$$

and since $v = u \sec \varphi$,

$$\frac{du}{d\varphi} = \frac{cu^4 \sec^4 \varphi}{g}$$

or

$$\frac{du}{u^4} = \frac{c}{g} \sec^4 \varphi \,.\, d\varphi.$$

Integrating,
$$\int_p^u \frac{du}{u^4} = \frac{c}{g}\int_\alpha^\phi \sec^4\varphi \,.\, d\varphi,$$
or
$$\frac{1}{u^3} - \frac{1}{p^3} = \frac{c}{g}(P_\alpha - P_\phi) \tag{19}$$
where
$$P_\alpha = 3\tan\alpha + \tan^3\alpha.$$

Putting $\varphi = \alpha - \psi$, equation (19) becomes
$$\frac{1}{u^3} - \frac{1}{p^3} = \frac{c}{g}(P_\alpha - P_{\alpha-\psi})$$
$$= \frac{c}{g}[3\tan\alpha + \tan^3\alpha - 3\tan(\alpha-\psi) - \tan^3(\alpha-\psi)].$$

Expanding the last two terms as functions of ψ by Maclaurin's Theorem, we have
$$\frac{1}{u^3} - \frac{1}{p^3} = \frac{c}{g}[3\sec^2\alpha.\psi + \ldots + 3\tan^2\alpha.\sec^2\alpha.\psi + \text{terms involving } \psi^2],$$
or
$$\frac{1}{u^3} - \frac{1}{p^3} = \frac{c}{g}[3\sec^4\alpha.\psi$$
$$+ \text{ terms involving squares and higher powers of } \psi].$$

To obtain an approximate value of ψ, we omit all terms containing its square and higher powers; whence
$$\psi = \frac{g\cos^4\alpha}{3c}\left(\frac{1}{u^3} - \frac{1}{p^3}\right). \tag{α}$$

Again, since q is the value of u when $\varphi = \beta$, we have from (α)
$$\alpha - \beta = \frac{g\cos^4\alpha}{3c}\left(\frac{1}{q^3} - \frac{1}{p^3}\right); \tag{β}$$
whence
$$\psi = \frac{\dfrac{1}{u^3} - \dfrac{1}{p^3}}{\dfrac{1}{q^3} - \dfrac{1}{p^3}}(\alpha - \beta). \tag{γ}$$

The X Integral.

In equation (7), putting $r = cv^3 = cu^3\sec^3\varphi$, we have
$$X = \frac{1}{c}\int_q^p \cos^2\varphi \frac{du}{u^2}. \tag{δ}$$
By Maclaurin's Theorem we have
$$\cos^2\varphi = \cos^2(\alpha - \psi) = \cos^2\alpha + \sin 2\alpha.\psi + \text{terms in } \psi^2; \tag{ϵ}$$

omitting the terms containing powers of ψ higher than the first and substituting in (δ),

$$X = \frac{\cos^2\alpha}{c}\int_q^p \frac{du}{u^3} + \frac{\sin 2\alpha}{c}\int_q^p \psi\frac{du}{u^3}.$$

Introducing the value of ψ from (γ), we have

$$X = \frac{\cos^2\alpha}{c}\int_q^p \frac{du}{u^3} + \frac{(\alpha-\beta)\sin 2\alpha}{c\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}\int_q^p \left(\frac{du}{u^6} - \frac{1}{p^3}\frac{du}{u^3}\right).$$

Whence, integrating,

$$X = \frac{\cos^2\alpha}{c}\left(\frac{1}{q}-\frac{1}{p}\right) + \frac{(\alpha-\beta)\sin 2\alpha}{c\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}\left[\frac{1}{4}\left(\frac{1}{q^4}-\frac{1}{p^4}\right) - \frac{1}{p^3}\left(\frac{1}{q}-\frac{1}{p}\right)\right],$$

and, putting $\frac{1}{q}-\frac{1}{p}=Q$,

$$X = \frac{Q}{c}\left\{\cos^2\alpha + \sin 2\alpha.(\alpha-\beta)\frac{\frac{1}{q^3}+\frac{1}{q^2p}+\frac{1}{qp^2}-\frac{3}{p^3}}{4\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}\right\}.$$

It is now necessary to obtain an approximate value of the fraction

$$f = \frac{\frac{1}{q^3}+\frac{1}{q^2p}+\frac{1}{qp^2}-\frac{3}{p^3}}{4\left(\frac{1}{q^3}-\frac{1}{p^3}\right)} = \frac{\frac{1}{q^2}+\frac{2}{qp}+\frac{3}{p^2}}{4\left(\frac{1}{q^2}+\frac{1}{qp}+\frac{1}{p^2}\right)} = \frac{p^2+2pq+3q^2}{4(p^2+pq+q^2)}.$$

Put $l=\frac{p-q}{p+q}$, whence $p=\frac{1+l}{1-l}q$. We now substitute this value of p in the expression for f and omit l^2 in the result, since l is a small quantity.

$$f = \frac{\left(\frac{1+l}{1-l}\right)^2 q^2 + 2\left(\frac{1+l}{1-l}\right)q^2 + 3q^2}{4\left[\left(\frac{1+l}{1-l}\right)^2 q^2 + \left(\frac{1+l}{1-l}\right)q^2 + q^2\right]}$$

$$= \frac{(1+l)^2 + 2(1-l^2) + 3(1-l)^2}{4[(1+l)^2+(1-l^2)+(1-l)^2]},$$

and, omitting l^3, we have approximately

$$f = \frac{6-4l+2l^2}{4(3+l^2)} = \frac{3-2l}{6}.$$

Whence $$X = \frac{Q}{c}\left\{\cos^2\alpha + \sin 2\alpha.(\alpha-\beta)\frac{3-2l}{6}\right\}. \qquad (\eta)$$

Comparing the equations (ϵ) and (η), it is evident that the expression in braces may be put equal to $\cos^2\bar{\varphi}$, if

$$\bar{\varphi}=\alpha-\frac{3-2l}{6}(\alpha-\beta)=\alpha-\frac{\alpha-\beta}{2}+\frac{l}{3}(\alpha-\beta)$$
$$=\frac{\alpha+\beta}{2}+\frac{p-q}{3(p+q)}(\alpha-\beta).$$

That is, we have

$$X=\frac{\cos^2\bar{\varphi}}{c}Q,\ Q=\frac{1}{q}-\frac{1}{p},\ \text{and}\ \bar{\varphi}=\frac{\alpha+\beta}{2}+\frac{p-q}{3(p+q)}(\alpha-\beta).$$

The Y Integral.

In equation (8), putting $r=cu^3\sec^2\varphi$ we have

$$Y=\frac{1}{c}\int_q^p\sin\varphi\cos\varphi\,\frac{du}{u^2}. \qquad (d)$$

Expanding by Maclaurin's Theorem, we have

$$\sin\varphi\cos\varphi=\sin(\alpha-\psi)\cos(\alpha-\psi)$$
$$=\sin\alpha\cos\alpha-\cos 2\alpha\,.\,\psi+\text{terms in }\psi^2.$$

Applying to this integral the same process we applied to (δ), we have

$$Y=\frac{\sin\bar{\varphi}\cos\bar{\varphi}}{c}Q,$$

in which $\bar{\varphi}$ and Q have the same values as in the X integral.

The Time Integral.

Putting $r=cu^3\sec^2\varphi$ in equation (6), we derive

$$dt=-\frac{du}{cu^3\sec^2\varphi\cos\varphi};\ \text{whence}\ T=\frac{1}{c}\int_q^p\cos^2\varphi\,\frac{du}{u^3}.$$

Substituting the value of $\cos^2\varphi$ from equation (ϵ),

$$T=\frac{\cos^2\alpha}{c}\int_q^p\frac{du}{u^3}+\frac{\sin 2\alpha}{c}\int_q^p\psi\,.\,\frac{du}{u^3}.$$

Integrating and introducing the value of ψ from the equation (γ)

$$T=\frac{\cos^2\alpha}{c}\,\frac{1}{2}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)+\frac{(\alpha-\beta)\sin 2\alpha}{c\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}\int_q^p\left(\frac{du}{u^6}-\frac{1}{p^3}\,\frac{du}{u^3}\right)$$
$$=\frac{\cos^2\alpha}{c}\,\frac{1}{2}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)+\frac{(\alpha-\beta)\sin 2\alpha}{c\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}\left[\frac{1}{5}\left(\frac{1}{q^5}-\frac{1}{p^5}\right)\right.$$
$$\left.-\frac{1}{2p^3}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)\right];$$

whence, putting $Q'=\frac{1}{2}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)$, we have

$$T=\frac{Q'}{c}\left\{\cos^2\alpha+\sin 2\alpha.(\alpha-\beta)\frac{\frac{1}{5}\left(\frac{1}{q^5}-\frac{1}{p^5}\right)-\frac{1}{2p^3}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)}{\frac{1}{2}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}\right\}.$$

Denoting by f' the fraction in the right hand term, and reducing,

$$f=\frac{2\left(\frac{1}{q^5}-\frac{1}{p^5}\right)-\frac{5}{p^3}\left(\frac{1}{q^2}-\frac{1}{p^2}\right)}{5\left(\frac{1}{q^2}-\frac{1}{p^2}\right)\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}$$

$$=\frac{2\left(\frac{1}{q^4}+\frac{1}{q^3p}+\frac{1}{q^2p^2}+\frac{1}{qp^3}+\frac{1}{p^4}\right)-\frac{5}{p^3}\left(\frac{1}{q}+\frac{1}{p}\right)}{5\left(\frac{1}{q}+\frac{1}{p}\right)\left(\frac{1}{q^3}-\frac{1}{p^3}\right)}$$

$$=\frac{2\frac{p^3}{q^3}+4\frac{p^2}{q^2}+6\frac{p}{q}+3}{5\left(\frac{p}{q}+1\right)\left(\frac{p^2}{q^2}+\frac{p}{q}+1\right)}=\frac{2p^3+4p^2q+6pq^2+3q^3}{5(p+q)(p^2+pq+q^2)}.$$

Putting $l=\frac{p-q}{p+q}$, whence $p=\frac{1+l}{1-l}q$, we have

$$f'=\frac{2\left(\frac{1+l}{1-l}\right)^3q^3+4\left(\frac{1+l}{1-l}\right)^2q^3+6\left(\frac{1+l}{1-l}\right)q^3+3q^3}{5\frac{2q}{1-l}\left[\left(\frac{1+l}{1-l}\right)^2q^2+\left(\frac{1+l}{1-l}\right)q^2+q^2\right]}$$

$$=\frac{2(1+l)^3+4(1+l)^2(1-l)+6(1+l)(1-l)^2+3(1-l)^3}{10[(1+l)^2+1-l^2+(1-l)^2]};$$

whence $f'=\frac{15-5l+5l^2+l^3}{10(3+l^2)}=\frac{3-l}{6}$ (omitting l^2 and l^3).

Hence $T=\frac{Q'}{c}\left\{\cos^2\alpha+\sin 2\alpha.(\alpha-\beta).\frac{3-l}{6}\right\}.$

On comparing this equation with (ϵ), it is obvious that we can express T in the form

$$T=\frac{Q'}{c}\cos^2\bar{\varphi}',\text{ if }\bar{\varphi}'=\alpha-\frac{3-l}{6}(\alpha-\beta)=\alpha-\frac{\alpha-\beta}{2}+\frac{l}{6}(\alpha-\beta),$$

or
$$\bar{\varphi}'=\frac{\alpha+\beta}{2}+\frac{1}{6}\frac{p-q}{p+q}(\alpha-\beta).$$

The equations and steps will now be written in the order of calculation.

Summary.

$\bar{\varphi}_1 = \frac{\alpha+\beta}{2}$ for low angles of departure, or $\tan\bar{\varphi}_1 = \frac{\tan\alpha + \tan\beta}{2}$ for high angles of departure.

If V is the muzzle velocity of the projectile, and α the angle of departure, then

$$p = V\cos\alpha, \text{ and } p\sec\bar{\varphi}_1 = V\cos\alpha\sec\bar{\varphi}_1;$$

then $q\sec\bar{\varphi}_1$ can be found from Table IV by equation (16), thus

$$D_{q\sec\bar{\phi}_1} = D_{p\sec\bar{\phi}_1} - \frac{d^2}{w} D\sec\bar{\varphi}_1,$$

when $D = \alpha - \beta$ in degrees.

The correct value of $\bar{\varphi}$ can be found for the ascending branch thus:

$$\bar{\varphi} = \bar{\varphi}_1 + \frac{1}{3}\frac{p-q}{p+q}(\alpha-\beta);$$

then X can be found from equation (12) thus:

$$X = \frac{w}{d^2}\cos\bar{\varphi}\,(S_{p\sec\bar{\phi}} - S_{q\sec\bar{\phi}})$$

and $$Y = \frac{w}{d^2}\sin\bar{\varphi}\,(S_{p\sec\bar{\phi}} - S_{q\sec\bar{\phi}}).$$

For the correct determination of the time

$$\bar{\varphi}' = \bar{\varphi}_1 + \frac{1}{6}\frac{p-q}{p+q}(\alpha-\beta);$$

but as the value of $\bar{\varphi}'$ occurs only as a limit of integration in the time integral, the value $\bar{\varphi}_1$ may approximately be used in its stead, and will give accurate results.

Then T can be found from equation (14) thus

$$T = \frac{w}{d^2}(T_{p\sec\bar{\phi}'} - T_{q\sec\bar{\phi}'}).$$

This process will be repeated for each component arc in the ascending branch, and a similar process is carried on for the descending branch—using the proper formulas for finding $\bar{\varphi}$ in that branch. The sum of all the X's in the component arcs gives the range, and the sum of all the T's gives the time of flight.

Evidently the sum of the Y's for the descending branch must equal the sum of all the Y's in the ascending branch, and if the former differs from the latter, they must be made to agree, by taking a new value for the angle β. If the difference is slight, the correction for range and time of flight may approximately be made, by assuming that the projectile moves, for the short distance near its fall, in a straight line, and the solution of a right triangle in which two parts are known is all that is necessary. In ordinary cases it will be sufficient to divide the whole trajectory into two arcs, but if great accuracy is desired, a further subdivision will be necessary.

Example.

An 8-inch projectile is fired at an angle of departure of 23°, with a muzzle velocity of 920 f. s. Weight of projectile, 180 pounds, $d = 8$ inches; find the range and time of flight. Divide the trajectory into two arcs, one ascending and the other descending.

For the ascending branch,

$$D = 23^\circ$$
$$V = 920 \text{ f. s.}$$
$$\frac{d^2}{w} = \frac{64}{180} = .35555,$$

then
$$\tan \bar{\varphi}_1 = \frac{\tan 23^\circ}{2} \quad \therefore \ \bar{\varphi}_1 = 11^\circ 59'.$$
$$p = 920 \cos 23^\circ = 846.9 \text{ f. s.}$$
$$p \sec \bar{\varphi}_1 = 846.9 \sec 11^\circ 59' = 865.7 \text{ f. s.}$$

Now
$$\frac{d^2}{w} D = \cos \bar{\varphi}_1 \left(D_{p \sec \bar{\phi}_1} - D_{q \sec \bar{\phi}_1}\right)$$

or
$$8.3599 = D_{865.7} - D_{q \sec \bar{\phi}_1}$$

therefore by Table IV,

$$D_{q \sec \bar{\phi}_1} = 66.7529$$
$$q \sec \bar{\varphi}_1 = 727.9 \text{ f. s.}$$
$$q = 712 \text{ f. s.}$$

We can now obtain a value of $\bar{\varphi}$ to employ in finding X_1 and Y_1.

$$\bar{\varphi} = \bar{\varphi}_1 + \frac{1}{3} \frac{p-q}{p+q} (a - \beta)$$
$$= 11^\circ 59' + \frac{846.9 - 712}{846.9 + 712} \times \frac{23^\circ \times 60}{3}$$
$$= 11^\circ 59' + 40' = 12^\circ 39'.$$

Now, $$X = \frac{\cos \overline{\varphi}}{\frac{d^2}{w}} S_{p \sec \overline{\phi}} - S_{q \sec \overline{\phi}}$$

$$= \frac{\cos 12^\circ 39'}{\frac{d^2}{w}} (S_{865.7} - S_{727.9}).$$ See Table II.

$$= 7784.5 \text{ feet.}$$

The values of $p \sec \overline{\varphi}$ and $q \sec \overline{\varphi}$ in the expressions $S_{p \sec \overline{\phi}}$ and $S_{q \sec \overline{\phi}}$ may be taken the same as in the calculation of D without appreciable loss of accuracy.

$$Y = X \tan \overline{\varphi} = 1747.2 \text{ feet}$$

and $$\frac{d^2}{w} T = T_{865.7} - T_{727.9}.$$ See Table III.

Whence $$T = 10.0806 \text{ seconds.}$$

For the descending branch assume an angle of descent of $27^\circ 54' = 27^\circ.9$, then

$$\tan \overline{\varphi}_1 = \frac{\tan 27^\circ 54'}{2}$$
$$= 14^\circ 49'.7.$$

The p of this arc is the q of the last arc $= 712$ f. s., and $p \sec \overline{\varphi}_1 = 736.6$ f. s.

Finding $q \sec \overline{\varphi}_1$ as before, we have

$$q \sec \overline{\varphi}_1 = 631.6 \text{ f. s.}$$

whence $$q = 610.6 \text{ f. s.}$$

The value of $\overline{\varphi}$ to employ with X and Y in the descending branch is:

$$\overline{\varphi} = \overline{\varphi}_1 - \frac{p - q}{p + q} \frac{\beta - a}{3}$$
$$= \overline{\varphi}_1 - \frac{712 - 610.6}{712 + 610.6} \times \frac{27.9 \times 60}{3}$$
$$= 14^\circ 49'.7 - 42'.7$$
$$= 14^\circ 7'.$$

Now, $$X = \frac{\cos 14^\circ 7'}{\frac{d^2}{w}} (S_{736.6} - S_{631.6})$$

$$= 7040.4 \text{ feet,}$$

and $$Y = X \tan 14^\circ 7'$$
$$= 1770.6 \text{ feet.}$$

Also,
$$T = \frac{T_{736.6} - T_{631.6}}{\frac{d^2}{w}}$$
$$= 10.6607 \text{ seconds.}$$

It will be noted that the projectile has fallen 23.4 feet more than it rose (1770.6—1747.2). If therefore the target is in the same horizontal plane as the muzzle of the gun, it will be necessary to apply a correction to the angle of fall assumed, and to find again for the descending branch the values of X, Y and T which will be given by the corrected angle of fall.

If we suppose, however, the projectile to move in a straight line inclined at 27° 54′ to the horizontal, and with a uniform velocity of which the horizontal component is 610.6 f. s. for the short distance considered, we shall have for the correction to apply to the range,

$$\text{Correction} = 23.4 \cot 27^\circ\ 54'$$
$$= 44.1 \text{ feet} = 14.7 \text{ yds.}$$

And the correction for the time of flight

$$= \frac{s}{v} = \frac{44.1}{610.6} = .0722 \text{ second.}$$

Therefore the true range is

$$4941.6 - 14.7 = 4926.9 \text{ yds.}$$

And the time of flight is

$$20.7413 - .0722 = 20.6691 \text{ sec.}$$

It must be remembered that the tables are computed for values of K which were determined by the use of projectiles of which the radius of the head is 1½ calibres; moreover, with the exception of the later experiments in 1879–80, studded projectiles were used. If then any other form of head is used, and if the conditions are such as give great smoothness of flight—as is probably the case with forced projectiles which are well centred—these tables will not give as accurate results as could be desired. It is difficult to determime just what correction should be applied, but in want of a more accurate plan it is suggested that one computed range be compared with an actual range determined by firing the gun once. The ratio of the observed range to the computed range will give some idea of the approximate correction to be applied to other computed ranges for the same gun and projectile.

The labor of computing trajectories by Niven's method will evidently be greatly diminished by the use of any method which will give a close approximation to the angle of fall. The following plan will give very good results when the angle of departure is not great, say 5°; and will give a better result, probably up to angles of departure of 10°, than can generally be obtained without large experience, by a simple guess.

Suppose the ascending branch of the trajectory to have been computed, and the height of the trajectory at vertex or Y_1 to have been found. Now, if we suppose the motion of the projectile in a vertical direction to be unresisted, and that gravity is the only force acting upon it in that direction, while passing over the descending branch, evidently we may find the approximate time it takes the projectile to fall to any vertical height less than the vertex.

We have,

$$S = \tfrac{1}{2} g t^2,$$

S being the height through which the projectile falls; and, if we are computing the range on the horizontal plane through the muzzle of the gun, is evidently equal to Y_1, and t is easily found; t being the approximate time it takes the projectile to pass over the descending branch.

Now, by the use of Table III, since we know the velocity of the projectile at the vertex and the time t, the approximate velocity at the point of fall can be determined.

This done, with the velocity at the vertex and the velocity at the point of fall as arguments, we enter Table IV, and the approximate angle of fall may be easily found.

Example.

A 3-in. B. L. R. projectile is fired at angle of departure of 2°35′, $d = 3''$, $w = 7$ lbs., muzzle velocity 1012 f. s. The ascending branch of the trajectory as computed gives $Y_1 = 29.805$ ft., and the horizontal velocity at the vertex, or $q_1 = 900$ f. s. As the range is required on the horizontal plane through the muzzle of the gun, we have

$$29.805 = \frac{32.2}{2} t^2;$$

whence

$$t = 1.361 \text{ seconds.}$$

Now,

$$\frac{d^2}{w} = \frac{9}{7}.$$

Hence, Table III,

$$\tfrac{8}{7}\,1.361 = 227.9544 - T_{v_2},$$

v_2 being the horizontal velocity at the point of fall, which we wish to find.

$$v_2 = 818 \text{ f. s.}$$

Now,
$$\frac{d^2}{w} D = D_{900} - D_{818}$$
$$\therefore \quad D = \tfrac{7}{8}\,[76.5005 - 72.7685],$$

whence $D = \beta = 2^\circ.903$, or $2^\circ 45'$, say. Using this value of the angle of fall in computing the descending branch of the trajectory, Y_2 is found to be 27.85 ft., or the shot is still 1.96 feet too high. Evidently no great error will be made in supposing the remainder of the trajectory to the point of fall to be a straight line.

Examples.

Example 1.—A 6-in. projectile leaves the gun at an angle of departure of 4°, with an initial velocity of 2100 f. s.; $w = 64$ lbs., $d = 6$ in. Find the range on horizontal plane through the muzzle of the gun, and time of flight (assume angle of fall about 6°30′).

Answer,

$$\text{Ascending branch} \begin{cases} X_1 = 5809.3 \text{ ft.} \\ Y_1 = 233.91 \text{ "} \\ T_1 = 3.5207 \text{ sec.} \end{cases}$$

$$\text{Descending branch} \begin{cases} X_2 = 4492 \text{ ft.} \\ Y_2 = 233.91 \\ T_2 = 3.9847 \text{ sec.} \end{cases}$$

Hence $R = 3434$ yards; $T = 7.51$ sec.

Example 2.—An 8-in. shot is fired at an angle of departure of 9° from a point 20 ft. above the water, find the range on the water and time of flight, $d = 8''$, $w = 180$ lbs., M. V. = 1350 f. s.

Ans. $R = 4041.8$ yds.
$T = 11.681$ sec.

Example 3.—What will be the range on the horizontal plane through the muzzle of the gun, of the projectile fired under the conditions of Example 2? Ans. $R = 4013$ yds.

Example 4.—A 3-in. projectile is fired at an angle of departure of 2°35′, $d = 3$, $w = 7$, M.V. = 1012. Required the range on the horizontal plane through the gun. Ans. 815 yds.

Example 5.—A projectile fired from the 60-pdr. B. L. R. leaves the gun at an angle of departure of 1°, $d = 5''.3$, $w = 46$ lbs. M. V. $=$ 1071 f. s. Required the range and time of flight.

Ans. $R = 403$ yds.
$T = 1.14$ sec.

Example 6.—What is the range of the projectile from the 60-pdr. B. L. R. when the angle of departure is 2°—the other conditions being the same as in Example 5? Ans. $R = 760$ yds.

Example 7.—The conditions being the same as in Example 5, with an angle of departure of 3°, what is the range?

Ans. $R = 1100$.

Example 8.—It is required to find how near a shot projected as in Example 5, will pass to a point 10 feet above the horizontal plane, and 403 yards horizontally from the muzzle of the gun, when the man using the gun assumes the trajectory rigid.

Ans. The shot will pass .42 foot too low.

CHAPTER VII.

Correction of the Fire of Guns for Wind, and Motion of Gun and Target.

Among the principal causes of variation in range and deviation in direction of flight of rifled projectiles, independent of the errors of the gun itself, are the influence of the wind and the motion of the gun. The motion of the target, if not allowed for, also affects accuracy of fire.

In the Ordnance Instructions for the U. S. Navy, Articles 766 to 772, are general instructions in regard to "corrections for speed, wind," etc., and in this chapter an attempt will be made to show how these instructions may be carried out in detail, how officers commanding divisions of guns may construct for their use simple tables, by which the sight-bar and sliding leaf may be set to allow for these corrections, leaving the gun captain free to point his gun directly at the target, and to fire when his line of sight and the point to be hit coincide.

In these days of high-powered guns and high speed of modern ships, much valuable time would be lost in trying to find the amount of the corrections necessary to apply to the range and direction of flight by trial shots alone, especially if the guns and target are in motion and if a ship's battery is composed of guns of different calibre and class; and it is believed also that with expensive ammunition, of which but a limited supply can be carried, owing to its weight and the space necessary for stowage, a great saving would result if approximate corrections were applied to the first trial shot, even if the plan was not followed thereafter.

The speed, distance and direction of motion of the target, and the force and direction of the wind, are subject to the errors of estimation or of observation in any case; and the estimations or observations having been made, the tables, if easy of application, furnish a quick and approximately accurate solution of what is otherwise attempted by a pure guess.

By practice officers can soon become very expert in the use of tables to apply corrections, and the time thus expended cannot be used to greater advantage if it will lead to any improvement in gunnery practice.

It must not be supposed that the formulae as finally reached furnish a rigid solution of the problems, but they give a close approximation to the truth, and will suffice for nearly all cases which are likely to happen in firing guns at sea.

Let us now consider in detail the three causes before mentioned.

The Influence of the Wind.

The method followed is that given by M. Hélie, Professeur à l'École d'Artillerie de la Marine, in a pamphlet on this subject published in 1874, somewhat abridged.

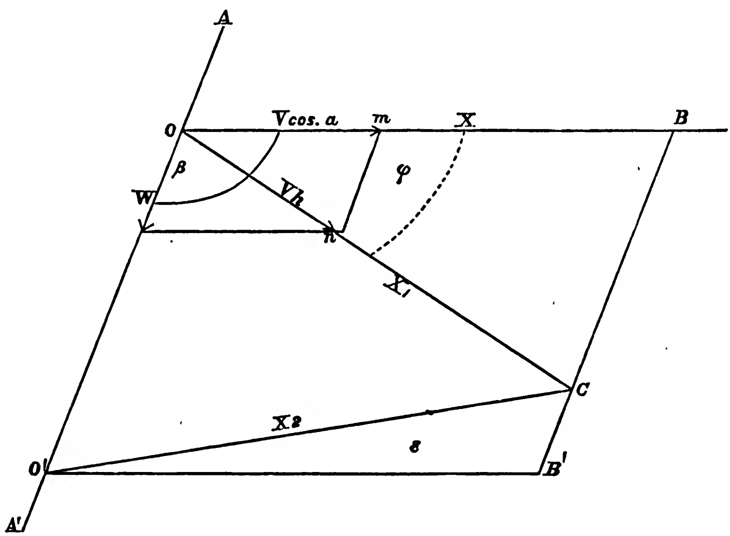


Fig. 1.

Let Fig. 1 represent the horizontal plane passing through the point O.

OB direction of line of fire.

B point of fall of the projectile when the gun is fired in a calm atmosphere and at an angle of elevation α, the gun being at O.

AA' direction of the wind with respect to the line of fire.

$V\cos\alpha$ horizontal component of the initial velocity V.

W velocity of the wind.

Didion, in his Traité de Balistique, shows that if we suppose the whole system—gun, projectile, atmosphere, etc.—to have a horizontal motion equal to and opposite to that of the wind, that is to say, if in the figure the wind is from $A'O$, if we suppose the whole system to have motion equal to that of the wind and in an opposite sense, the absolute velocity of the wind reduces to zero, and, so far as the relative positions of the parts of the system are concerned, we would have the condition of a calm atmosphere. On this hypothesis we suppose the system to have at the instant of firing a velocity equal to W and toward A' from O, then the projectile on leaving the gun has a resultant velocity of which the horizontal component is V_h, the direction of which is the resultant of $V\cos\alpha$ and W.

At the end of a given time T the projectile will fall at some point C, but the points O and B at the end of the same time have, owing to the motion given to the system, moved to the new positions O' and B', the line OB' parallel to OB, and the relative positions of the gun, point of fall and line of fire will be represented by the triangle $O'CB'$.

The time T, when a very close approximation is desired or when the angle of elevation is large, should be found from a formula in which $T = F(V_1, \alpha_1)$, in which V_1 is the resultant initial velocity. The movement of the system has not changed the vertical component of the initial velocity V and is always $V\sin\alpha$, hence these quantities may be found from the formula $V_1^2 = V^2\sin^2\alpha + V_h^2$, and

$$\tan\alpha = \frac{V\sin\alpha}{V_h}.$$

In addition the range X_1 corresponding to (V_1, α_1) should be found.

However, for the purpose of correcting firing at sea, at angles of elevation of 5° and less, which with the new guns will include ranges to at least 4000 yds. and with the 8-in. Converted Rifle to 2800 yds., distances within which naval actions will probably be fought, the error made by taking T and X as the time of flight and range respectively corresponding to the initial velocity V and elevation α will be very small and may be neglected. On this hypothesis we have, returning to the triangle $O'CB'$,

$$B'C = BB' - BC = WT - W\frac{X}{V\cos\alpha}$$

because $\dfrac{BC}{W} = \dfrac{X}{V\cos\alpha}$ from the similar triangles Omn and OBC.

Now
$$\frac{\sin \varepsilon}{\sin \beta} = \frac{CB'}{O'C} = \frac{WT - W\dfrac{X}{V\cos a}}{O'C}$$

or
$$X_2 \sin \varepsilon = \left(T - \frac{X}{V\cos a}\right) W \sin \beta.$$

Evidently $X_2 \sin \varepsilon$ is the lateral deviation of the point C from the line of fire $O'B'$; and since ε is a very small angle, its sine is sensibly equal to the tangent, and we have

$$\text{deviation} = d = \left(T - \frac{X}{V\cos a}\right) W \sin \beta, \qquad (1)$$

which is the formula given in Breger's "Probability of hitting an object of any form," translated by Lieutenant C. A. Stone, U. S. N.; page 50.

From the triangle OCO' we have

$$X_2 = X_1^2 + W^2T^2 - 2X_1 WT \cos(\beta - \varphi).$$

In this equation the term W^2T^2 is so small when compared with X_1^2 that it may be neglected or multiplied by a quantity less than unity; $\cos^2(\beta - \varphi)$, say, and the expression becomes a perfect square; we have then,
$$X_2 = X_1 - TW \cos(\beta - \varphi),$$

and since φ is always a very small angle and becomes inappreciable when β is small, we can without great error write $\cos \beta$ for $\cos(\beta - \varphi)$, and the formula becomes

$$X_2 = X_1 - WT \cos \beta.$$

The time T and X_1 correspond, as has been before explained, to the initial velocity V_1 and the angle of elevation a_1, but for the purpose of approximations affecting firing at sea, within the limits before mentioned, we may use T and X corresponding to V and a, and we have finally
$$X_2 = X - WT \cos \beta.$$

The angle β is considered as a positive angle, and is reckoned from the target on either side of the line of fire around to the rear of the gun.

From (1) and (2) we see that the sign of d is always positive, and that the sign of the second term of the second member of (2) is negative for values of β between 0° and 90° and positive for values between 90° and 180°.

Colonel Maitland, R. A., gives a method for finding the lateral deviation due to the wind, less simple in form than the preceding, as follows; the wind being considered as a constant force blowing across the range, and acting uniformly on the projectile. His method can be used if considered desirable.

Let $p =$ pressure of wind in pounds per square foot,

$A =$ area of longitudinal section of projectile in square feet.

From the computed results of many practice tables he concludes that the expression pA represents with practical accuracy the pressure on the projectile tending to move it from the direction of the line of sight.

Sir Henry James gives the following ratio of pressure to velocity of the wind:

$$p = .005\, V^2 \text{ in miles per hour,}$$

$$\text{or } p = .00232438\, V^2 \text{ in feet per second.}$$

Now if this is a uniform pressure, and if the wind makes an angle D with the line of fire, the path of the projectile will resemble a parabola, and for heavy shot and short ranges or light winds the formula is:

$$d = \text{deviation in feet} = \frac{p \sin DAgt^2}{2\omega},$$

ω being the weight of the projectile.

His formula may be obtained as follows: Let $M =$ mass of the projectile, and $f =$ accelerating force acting on the projectile, then

$$Mf = p \sin DA$$

or

$$f = \frac{pAg \sin D}{W} = \frac{dv}{dt}$$

integrating

$$V = \frac{pAg \sin Dt}{W} + C,$$

when $t = 0$, $V = 0$ $\therefore$ $C = 0$

$$V = \frac{ds}{dt} = \frac{pAg \sin Dt}{w},$$

integrating

$$S = \frac{p \sin DAgt^2}{2w} + C'$$

when $t = 0$, $S = 0$ $\therefore$ $C' = 0$

hence

$$S = d = \frac{p \sin DAgt^2}{2w}.$$

This equation is not accurate enough for light shot and high winds, since a light shot takes up a sideways velocity due to the wind more easily than a heavy one, and ultimately the shot will have the same velocity in that direction as the wind.

It is clear then, that the pressure of the wind on the side of the shot at any instant is that due to the difference between the sideways velocity of the latter and the velocity across the range of the former.

Colonel Maitland thus treats this case: putting V as the sideways velocity of the projectile communicated to it by the wind, we have $W \sin D - V$ as the difference before mentioned, and hence to find the pressure we have

$$p = .00232438\,(W \sin D - V)^2,$$

from the preceding we have

$$f = \frac{dv}{dt} = \frac{pAg}{w} = \frac{.00232438Ag}{\omega}\,(W \sin D - V)^2$$

$= a\,(W \sin D - V)^2$, when a is a constant for each nature of projectile.

$$\frac{dt}{dv} = \frac{1}{a}\left(\frac{1}{(W \sin D - V)^2}\right);$$

integrating
$$t = \frac{1}{a} \cdot \frac{1}{W \sin D - V} + C$$

when $t = 0$, $V = 0$ $\therefore$ $C = -\dfrac{1}{a W \sin D}$.

Solve for V, we have,

$$V = W \sin D - \frac{W \sin D}{at W \sin D + 1} = \frac{ds}{dt}$$

integrating, we obtain

$$S = W \sin Dt - \frac{1}{a} \cdot \log_e (at W \sin D + 1) + C'$$

when $t = 0$, $S = 0$ $\therefore$ $C' = 0$.

Substituting the value of a, we have for deviation in feet,

$$d = W \sin Dt \, \frac{\omega}{.00232438Ag} \cdot \log_e\left(\frac{.00232438Ag\,W \sin Dt}{\omega} + 1\right).$$

In this formula the first term gives the distance passed over by the wind in the time t, and the second term the space lagged behind by the projectile in that time, and the difference is evidently the sideways travel of the shot.

This last remark applies to the formula deduced after Prof. Hélie's method.

In log-books kept at sea, a table of the force of the wind, pressure in pounds per square foot, corresponding to different velocities of the wind in miles per hour, is given, as follows. The velocity of the wind

is measured by an anemometer. It will be found that the pressure agrees very closely with results deduced from Sir Henry James' formula.

Nautical Scale.	Force of the Wind.	Pressure in Pounds per Square Foot.	Velocity of the Wind. Miles per Hour.
0	Calm..................................		
1	Light Airs.	0.004 to 0.019	1 to 2
2	Light Breezes........................	0.08	4
3	Gentle Breezes......................	0.4	9
4	Moderate Breezes..................	1.0	14
5	Stiff Breeze..........................	1.5	17
6	Fresh Breeze.........................	2.0	20
7	Very Fresh Breeze................	3.0	24
8	Moderate Gale......................	5.0	30
9	Strong Gale...........................	8.0	40
10	Very Strong Gale..................	23.0	67
etc.	Etc....................................	etc.	etc.

The Motion of the Gun.

In this case the projectile as it leaves the gun is supposed to have a velocity equal to and in the direction of the motion of the gun itself. Now if we can consider the motion in this direction as practically unresisted by the air, an approximate solution is very easily reached.

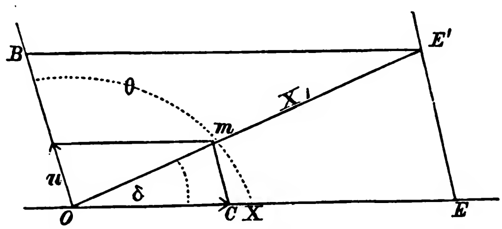


Fig. 2.

On the above hypothesis consider Fig. 2 as the horizontal plane through the point O at which the gun is situated. Suppose the gun to be moving toward B with a velocity U, and if OE is the line of fire, E being the point of fall of the projectile with an initial velocity V and angle of elevation α when the gun is stationary. And if X is the range and T the time of flight corresponding to these data, and if the projectile during the time T has a uniform velocity in the direction of OB equal to U, then at the end of the time T it will fall at E'; EE' being equal and parallel to $OB = UT$.

The time T and range X can be found from formulae in which they occur as functions of V. α, or may be taken from range tables.

For ranges, then, at angles of elevation less than 5°, we may with close approximation take T for a range X that would be given for an initial velocity V and angle of elevation α.

Thus approximating, we have from the triangle $BE'O$, the line $BE' = X$.

$$X_1^2 = X_2^2 + U^2T^2 + 2UTX \cos \theta.$$

But U^2T^2 is very small when compared with X_2^2 and may be neglected or multiplied by a quantity less than unity, $\cos^2\theta$, say, and we have

$$X_1 = X + UT \cos \theta. \tag{3}$$

From the similar triangles OCm and OEE' we have

$$\frac{\sin \delta}{\sin (\theta - \delta)} = \frac{U}{V \cos \alpha} = \frac{UT}{X}$$

or $X \sin \delta = UT \sin (\theta - \delta)$.

Since δ is always a very small angle and vanishes when $\theta = 0$, we may write $\sin \theta$ for $\sin (\theta - \delta)$. Again the sine and tangent of δ are sensibly equal, and if the lateral deviation is $X \tan \delta$, we have

$$d = UT \sin \theta \tag{4}$$

From formulas (3) and (4) we see that the sign of the second term of the second member of (3) is positive for values of θ between 0° and 90° and negative for values between 90° and 180°; θ being taken as a positive angle and reckoned from the line of fire on either side around to the rear of the gun. d is always positive.

The Motion of the Target.

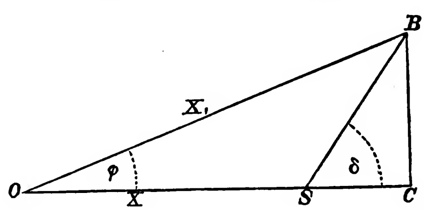


Fig. 3.

Let Fig. 3 be the horizontal plane through OaS.

If the target is at S, Fig. 3, and moving with a velocity U toward B, the line SB making an angle δ with the line of fire OS, the gun being fixed at O. At the end of the time T (T being the time for a

shot to travel the range X or OS), the target will have arrived at some point E and $BS = UT$.

Now the gun must be so aimed that the shot will fall at B at the moment the target arrives at that point, hence the sight-bar must be set for a range X_1 and the sliding leaf to allow for a lateral deviation BC at the range X_1. We have $BC = UT \sin \delta$ and $Sc = UT \cos \delta$, and since φ is always a small angle we have approximately

$$X_1 = X + UT \cos \delta \qquad (5)$$
$$d = UT \sin \delta \qquad (6)$$

If d' is the lateral deviation for the range X it will be found practically to differ but little from d, and may be used in its stead.

d is always positive, and the second term of the second member of (5) is positive for values of S between 0° and 90°, and negative for values between 90° and 180°, δ being taken as a positive angle and reckoned from the line of fire on either side around to the rear of the gun.

The formulae may now be collected as follows:

$$\text{Influence of the wind} \begin{cases} X_2 = X - WT \cos B & (1) \\ d = \left(T - \dfrac{X}{V \cos \alpha}\right) W \sin \beta & (2) \end{cases}$$

$$\text{Motion of the gun} \begin{cases} X_1 = X + UT \cos \theta & (3) \\ d = UT \sin \theta & (4) \end{cases}$$

$$\text{Motion of the target} \begin{cases} X_1 = X + UT \cos \delta & (5) \\ d = UT \sin \delta & (6) \end{cases}$$

An inspection of the formulae shows that it will be necessary to construct one table showing the corrections to be made for the influence of the wind, and that one will suffice to show the corrections to be made for the motions of the gun and target. Each class of gun will require its own set of tables.

The plan is evident, and is simply to assume ranges varying from each other as much as may seem desirable; the lowest range probably not less than 1000 yards, and the highest not greater than that given by the gun with 5° elevation. The initial velocity is given in the Ordnance Instructions, where range tables are also to be found.

Then by formulas (1) and (2) compute for variations of 15° of the angle β, and with a fixed velocity of the wind, say 10 miles per hour, the values of the corrections in range and lateral deviation due to the wind, using the times of flight corresponding to the ranges assumed.

The value of the velocity of the wind is suggested at 10 miles an hour, since for any other values the corrections are very easily found, when once they are computed for that assumed velocity, and the labor of computation is lessened. For the corrections due to the motion of the gun and target, the process is the same, except that the velocity of the gun and target should be taken in knots, of 6080 feet each, per hour. When suitably arranged, the amounts of the corrections in range and lateral deviation are found by inspection and interpolation from the two tables for the range at which the gun is to be fired; the other arguments being the angle made by the direction of the wind or direction of motion of the gun and target with the line of fire, and the velocities of the wind, gun and target.

It is suggested that the tables be not made too elaborate, so as to require too much time in searching for the desired corrections. Once determined, the correction for range is applied to the sight-bar with its proper sign, and the correction for lateral deviation to the sliding leaf, or rather they are set to allow for them, and the latter requires a small table to use in connection with it.

The sliding leaf on breech sights is usually graduated in minutes of arc, and the divisions mean simply this, that the distance spaced off on the leaf for a division marked for, say 40′, is the tangent of an angle of 40° when the radius of the circle is the distance between the front and breech sights. Evidently the distance spaced off is found by taking the circular measure of the small angle in question, the radius of the circle being as before stated. The sine and tangent of so small an angle are sensibly equal to the length of the arc itself. The lateral deviation at any distance, due to the sliding leaf being moved to the 40′ mark, is found in exactly the same way; for example, the radius of the circle is the given range, then the deviation in yards is the circular measure of the small angle given, or the tangent of that angle when the radius of the circle is the range in yards.

If the sliding leaf is not graduated to minutes of arc there is no difficulty, since the distance of any division from the zero mark may be transferred to a scale by a pair of dividers, and we have then two similar triangles from which the lateral deviation due to the sliding leaf being moved any distance may be found for any assumed range.

For illustration, the necessary tables are computed for the 8-in. Converted Rifle, initial velocity 1450 f. s., and the times of flight corresponding to the assumed ranges are taken from the Ordnance Instructions, Range Table for this gun, p. 410.

The ranges are assumed at 1000, 1500, 2000, 2500 and 2800 yards; a greater number, differing less from each other, could be taken with additional labor and space occupied.

In using formula (2) it will be found that V may be used for $V \cos \alpha$ for angles less than 5°. For instance, 5° elevation the range of the 8-in. gun is 2817 yards $= 8451$ feet. We have $\frac{8451}{1450 \cos \alpha} = 5.85$ and $\frac{8451}{1450} = 5.83$, the difference between the two values may be neglected.

Formula (2) then becomes

$$d = \left(T - \frac{X}{V}\right) W \sin \beta.$$

TABLE I.

FOR INFLUENCE OF WIND. 8-in. M. L. R. I. V. 1450 f. s.

R = correction in range, yards. D = correction for lateral deviation, yards.

Velocity of Wind.	Time. Seconds	Range. Yards.	Angle made by direction of wind with line of fire.													
			0° or 180°		15° or 165°		30° or 150°		45° or 135°		60° or 120°		75° or 105°		90°	
			R	D	R	D	R	D	R	D	R	D	R	D	R	D
10 Miles per Hour.	2.23	1000	10.8	0	10.5	.2	9.4	.4	7.7	.5	5.4	.7	2.8	.57	0	0.8
	3.48	1500	17.0	0	16.4	.5	14.7	.9	12.0	1.3	8.5	1.6	4.4	1.8	0	1.9
	4.81	2000	23.5	0	22.7	.8	20.4	1.6	16.6	2.3	11.8	2.8	6.1	3.2	0	3.3
	6.23	2500	30.5	0	29.4	1.3	26.4	2.6	21.5	3.6	15.2	4.4	7.9	4.9	0	5.1
	7.11	2800	34.8	0	33.6	1.6	30.1	3.2	24.6	4.5	17.4	5.5	9.	6.1	0	6.4
20 Miles per Hour.	2.23	1000	21.6	0	21.0	.4	18.8	.8	15.4	1.1	10.8	1.3	5.6	1.5	0	1.6
	3.48	1500	34.0	0	32.8	1.0	29.4	1.9	24.0	2.6	17.0	3.2	8.8	3.6	0	3.8
	4.81	2000	47.0	0	45.4	1.7	40.8	3.3	33.2	4.6	23.6	5.7	12.2	6.3	0	6.6
	6.23	2500	61.0	0	58.8	2.6	52.8	5.1	43.0	7.3	30.4	8.9	15.8	9.9	0	10.2
	7.11	2800	69.6	0	67.2	3.3	60.2	6.4	49.2	9.0	34.8	11.1	18.0	12.4	0	12.8
30 Miles per Hour.	2.23	1000	32.4	0	31.5	.6	28.2	1.2	23.1	1.7	16.2	2.0	8.4	2.3	0	2.4
	3.48	1500	51.0	0	49.3	1.5	44.2	2.8	36.1	3.9	25.5	4.8	13.2	5.4	0	5.7
	4.81	2000	70.6	0	68.2	2.5	61.1	4.9	49.9	6.9	35.4	8.5	18.3	9.5	0	9.9
	6.23	2500	91.5	0	88.3	4.0	79.2	7.7	64.7	10.9	45.6	13.3	23.7	14.9	0	15.3
	7.11	2800	04.3	0	100.8	5.0	90.3	9.6	73.7	13.6	52.2	16.6	27.0	18.6	0	19.2

TABLE II.

Corrections for Motion of Gun or Target.

8-in. M. L. R., I. V. 1450 f. s. R = correction for range, in yards.
D = correction for deviation, in yards.

Speed of Gun or Target Knots.	Yards Range.	δ or θ 0° or 180°		δ or θ 15° or 165°		δ or θ 30° or 150°		δ or θ 45° or 135°	
		R	D	R	D	R	D	R	D
5	1000	6.3	0	6.0	1.7	5.4	3.1	4.4	4.4
	1500	9.8	0	9.4	2.6	8.5	4.9	6.9	7.0
	2000	13.5	0	13.0	3.5	11.7	6.8	9.6	9.6
	2500	17.5	0	16.9	4.5	15.2	8.8	12.4	12.5
	2800	20.0	0	19.3	5.2	17.3	10.0	14.1	14.1
8	1000	10.0	0	9.7	2.7	8.6	5.0	7.0	7.0
	1500	15.7	0	15.1	4.1	13.6	7.9	11.1	11.1
	2000	21.7	0	20.9	5.6	18.7	10.9	15.3	15.3
	2500	28.1	0	26.8	7.2	24.3	13.1	19.9	19.9
	2800	32.0	0	30.9	8.3	27.7	16.0	22.6	22.6
10	1000	12.5	0	12.1	3.3	10.8	6.3	8.8	8.8
	1500	19.6	0	18.9	5.1	17.0	9.8	13.9	13.9
	2000	27.1	0	26.1	7.0	23.4	13.6	19.2	19.2
	2500	35.1	0	33.8	9.1	30.4	17.6	24.9	24.9
	2800	40.0	0	38.6	10.4	34.6	20.0	28.3	28.3
12	1000	15.0	0	14.5	4.0	13.0	7.5	10.6	10.6
	1500	23.5	0	22.7	6.1	20.4	11.8	16.7	16.7
	2000	33.6	0	31.3	8.4	28.1	16.3	22.0	22.0
	2500	42.1	0	40.5	10.9	36.5	21.1	30.0	30.0
	2800	48.0	0	46.3	12.5	41.5	24.0	34.0	34.0
13	1000	16.2	0	15.7	4.3	14.0	8.2	11.4	11.4
	1500	25.5	0	24.5	6.6	22.1	12.7	18.1	18.1
	2000	35.2	0	33.9	9.1	30.4	17.7	25.0	25.0
	2500	45.6	0	43.9	11.8	39.5	22.9	32.4	32.4
	2800	52.0	0	50.2	13.5	45.0	26.0	36.8	36.8
14	1000	17.5	0	16.9	4.6	15.1	8.8	12.3	12.3
	1500	27.4	0	26.5	7.1	23.8	13.7	19.4	19.4
	2000	37.9	0	36.5	9.8	32.7	19.0	26.9	26.9
	2500	49.1	0	47.3	12.7	42.5	24.6	34.8	34.8
	2800	56.0	0	54.0	14.6	48.4	28.0	39.6	39.6
15	1000	18.8	0	18.1	5.0	16.2	9.4	13.2	13.2
	1500	29.4	0	28.3	7.7	25.5	14.7	20.8	20.8
	2000	40.6	0	39.1	10.5	35.1	20.4	28.8	28.8
	2500	52.6	0	50.7	13.6	45.6	26.4	37.3	37.3
	2800	60.0	0	58.0	15.6	51.9	30.0	42.4	42.4
20	1000	25.0	0	24.2	6.6	21.6	12.6	17.6	17.6
	1500	39.2	0	37.8	10.2	34.0	19.6	27.8	27.8
	2000	54.2	0	52.2	14.0	46.8	27.2	38.4	38.4
	2500	70.2	0	67.6	18.2	60.8	35.2	49.8	49.8
	2800	80.0	0	77.2	20.8	69.2	40.0	56.6	56.6
Speed of Gun or Target Knots.	Yards Range.	D	R	D	R	D	R	D	R
		δ or θ 90°		δ or θ 75° or 105°		δ or θ 60° or 120°		δ or θ 45° or 135°	

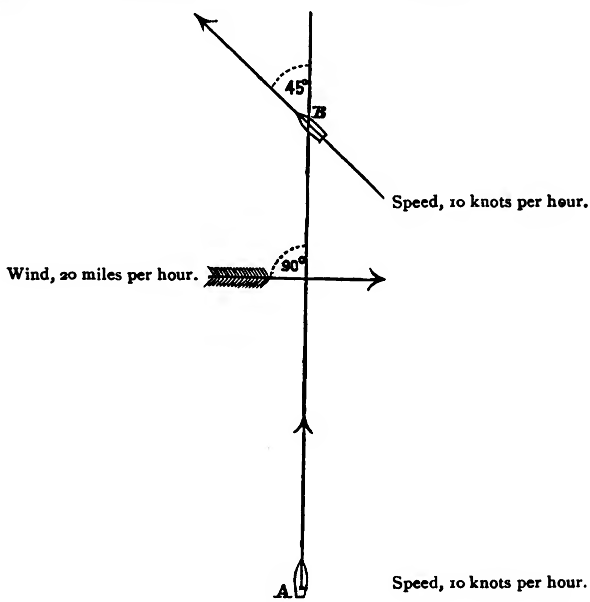
TABLE III.

Lateral Deviation in Yards for each Division on Sliding Leaf.

8-in. M. L. R. Distance between Sights 44".5.

Range Yards.	10'	20'	30'	40'	50'	1°
1000	2.9	5.8	8.7	11.6	14.5	17.4
1200	3.5	6.9	10.5	14.0	17.5	21.0
1400	4.1	8.2	12.3	16.4	20.5	24.6
1600	4.6	9.3	13.8	18.4	23.0	27.6
1800	5.2	10.4	15.6	20.8	26.0	31.2
2000	5.8	11.6	17.4	23.3	29.1	34.8
2200	6.4	12.8	19.2	25.6	32.0	38.4
2400	7.0	14.0	21.0	28.0	35.0	42.0
2600	7.6	15.2	22.8	30.4	38.0	45.6
2800	8.2	16.3	24.4	32.6	40.7	49.2

Example.—Suppose a ship at A firing with an 8-in. M. L. R. at a ship at B, which is steering a course making an angle of 45° with the line of fire; A at the instant of firing steering directly towards B and making 10 knots an hour. Wind across the range as indicated in the figure. Distance from A to B, 1500 yards.



From Table I the lateral deviation due to the wind is found to be 3.8 yards.

From Table II the correction for speed of A is -19.6 yards in range. Correction for speed of B, $+13.9$ yards in range, and 13.9 yards lateral deviation. The gun, then, must be pointed to the left of the line of fire to allow for a lateral deviation of $3.8+13.9=17.7$, say 18 yards, and the sliding leaf should then be set to the left at about the 40′ mark; since from Table III it is seen that that distance on the sliding leaf will cause about the deviation required at 1500 yards range. The range is not materially changed; it is 1494 yards, about.

The table showing the effect of the wind is constructed after Professor Hélie's formulas, as previously deduced, and the lateral deviations seem to be large. It is thought, however, that the results will agree fairly with experiment, but until extended experiments shall be made to establish the law of the deviation of projectiles due to the effect of the wind, any formulas are subject to some doubt. It is admitted that the tables may, perhaps, be more conveniently arranged than has been indicated, and, doubtless, ingenious officers who care to form and use such tables will devise some better plan of arrangement.

A set of tables, if they serve no other purpose, will give an excellent idea of the magnitude of the correction that should be applied in some way; and, if studied, must lead to an improvement in accuracy of fire.

When a ship is turning quickly, the influence of the *angular* velocity given to the gun on the lateral deviation of projectiles is sometimes a very appreciable quantity. If the lock-string is pulled at the moment the gun-captain decides the line of sight bears on the object to be hit, a short interval of time elapses for the charge to be ignited and the projectile to leave the muzzle of the gun. This interval of time evidently varies with the character of the firing apparatus, the powder, the length of bore or travel of the projectile, and other causes, and it is thought that in many cases may equal, if not exceed, .2 of a second. From experiments made at the Naval Academy, with the Schultz Chronograph, it was found that with the Navy percussion-locks the interval between the gun-captain throwing himself back on the lock lanyard and the explosion of the primer was 0.13 second, and to this must be added the time for the charge to be ignited and the shot to leave the bore.

Taking the case of a ship which completes a full circle in 4 minutes as an example, her angular velocity is $1°30'$ per second; and if the

interval of time from the moment the gun-captain decides to fire, to the moment the shot leaves the bore, is 0.2, the ship will swing through an arc of 18′ in that time; hence the lateral train of the gun is changed about that much, and by reference to the table for the sliding leaf of the 8-inch M. L. R., this would cause a lateral deviation of 44 feet at a range of 2800 yards. Except at long ranges and when the turning circle is quickly completed, the influence of this source of lateral deviation will probably not be great, especially if a quick-acting firing apparatus be used, and the gun-captains are trained to fire without any delay the instant the sights bear.

CHAPTER VIII.

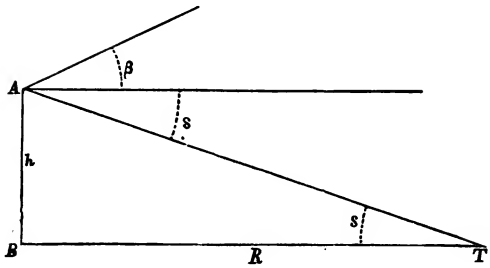
On the Preparation and Use of Range Tables.

§ I.

A range table should contain all the information which is necessary in order to use effectively any given projectile. Since, in the use of naval guns, the angles of projection and sight (see Chapter I for definitions of terms here used) are generally small, the trajectory may be assumed sensibly rigid; or, in other words, the sight bar may be used without regard to what may be the (small) elevation or depression of the target.

If the ranges are to be computed for various angles of departure, by the methods indicated in Chapter VI, it is evident that the jump angle must be known; since without this the angles of elevation cannot be found at all. This being known, successive values of the angle of departure $1°$, $2°$, $3°$, etc., are assumed, and the ranges, times of flight, greatest ordinates, and angles of fall corresponding are found. This method will give good results for projectiles of a form similar to those from which the tables were made; but, if the cost of firing a gun a large number of times is not too great, or if some other question of convenience does not forbid it, it is better to form the range table by experiment.

To this end the gun would be layed a number of times at successive angles of inclination by a spirit level and fired; and the mean ranges found be called the range due to this angle. If the firing take place over the land, the points of fall would be found accurately by measurement; but if it takes place over water, these points must be observed by two plane tables, one at the gun and the other at a point which will give the best cut. In either case, the planes on which the ranges are measured would probably not be the horizontal plane through the gun, the plane to which the ranges must be referred; and we must apply to the angle at which the gun was laid by spirit level, a correction such that the resulting angle shall be the angle of elevation. In the figure, let A be the position of the gun at a height h above



BT, the plane on which the range R is measured; let T be the point struck, and B be vertically under A. It is to be remembered that in all actual cases R is very great compared with h. The angle β would be that given by the spirit level; and since AT is the line of sight, $\beta + s$ is the angle of elevation. Also

$$\tan s = \frac{h}{R}$$

The observation is then recorded that, with an angle of elevation $\beta + s$, a range R was found. Obviously, if we slowly raise the sight bar, if one had been fitted before the gun was ranged, until we see the point T across the sights, we may put a scratch across its face and write the value of R on it. As, however, the bars are usually marked in even hundreds of yards, this would not be the best way to proceed.

If possible, the experimental firing should take place in standard conditions of barometer and thermometer, and the ranges and deflections corrected for the effect of the wind by methods given in Chapter VII.

Suppose, then, that a number of shots had been fired within the limits within which it was intended to establish the range table, proceeding by increments of 2° say, and that these had been corrected. In order to find the permanent angle i (see figure in Chapter I), it is usual to take the deflections at some considerable angle of projection, to compute its value (see figure already mentioned). If it should be found that the permanent angle so computed at a selected angle does not properly correct for the observed deflections at other angles, the sight bar might be left vertical, and a table constructed showing the amount of lateral motion to give the sight notch at each angle.

There will remain, since the correct determination of the permanent angle will cause the shot always to fall in the vertical plane containing

the line of sight, a series of observations connecting angles of elevation ($\beta + s$; the angle s is sometimes called the *additional* angle) with range. Since, in the general case, R is sensibly equal to $\sqrt{R^2 + h^2}$, we may record the former as the range. Also, in the sight-bar triangle (ABC, Chapter I), the angle BAC is the angle of elevation; and since AB is known, being the distance between sights; we may, having any number of relations connecting range and elevation, find the corresponding ones connecting range and sight-bar height. In case the bar is set at a permanent angle i, we may obviously (see figure, Chapter I) mark the line BC' which represents the sight-bar in this case.

We may then, to mark the sight-bar and determine the range table completely from the data we have, take either angles of elevation and ranges, or sight-bar heights and ranges. If we take the former, we might proceed as described on pp. 141–143, Text-Book of Ordnance and Gunnery. Or else, assuming a proper analytical form for the law connecting angles of elevation and range, compute the constants in the assumed equation by the Method of Least Squares. The latter method, when carefully chosen, probably involves less labor than the former, and is certainly more satisfactory and accurate.

The form of the relation connecting angles of elevation and range is not known. In a non-resisting medium it is

$$\sin 2\alpha = \frac{g}{V^2} R;$$

and this indicates that we might hope to get good results by assuming some such form as

$$\sin 2\alpha = aR + bR^2;$$

since the true form is doubtless more complex. For small angles (up to 5° or 6°) a more convenient form for computation would be

$$\alpha = aR + bR^2.$$

In the use of this, by taking α in minutes of arc, it would be a fairly large quantity, and a and b would not be so small as they would be if we took $\sin 2\alpha$. In case there is jump, and there always is, an absolute term must be inserted; since α, being the angle of elevation, is not in this case zero when R is zero. We may then finally adopt the form

$$\alpha = a + bR + cR^2.$$

In this equation, the jump angle gives a at once; since when $R = 0$, $a = -$jump angle. Therefore, writing it in the form

$$\frac{a - a}{R} = b + cR,$$

we have only b and c to determine by least squares.

§ II.

As an example of the foregoing, an account of the experimental ranging of the 3-inch steel B. L. R. No. 34, which was carried out in the winter of 1884–5 by the cadets of the first class at the Naval Academy, is here given.

The gun was on the boat carriage, and was held by a vertical bolt and chain attached to the front end of the slide. It was placed on the sea wall, behind the gymnasium, and the fire was directed at different buoys and marks in the bay, as these gave clear ranges. The fall of the projectiles was marked by two plane tables, one in a window of the gymnasium almost over the gun, and the other at a suitable point to give a good cut according to the range to be determined. The angles so found gave, by means of a piece of tracing paper and an enlarged chart on which were plotted all the stations occupied, the ranges and deflections over the water. Thus was reached data which may be summarized as in the following table:

No. of shots fired.	Angles of elevation ($\beta + s$).	Time of flight.	Barometer.	Mean Ranges. Yards.	Wind. Feet per minute.
6	16′	$1^s.62$	29.97	430	1468 f. m.
4	2°..07′	$3^s.62$	29.97	1003	do.
3	4°..05′	$5^s.42$	29.97	1472	400 f. m.
5	6°..04′	$7^s.40$	29.97	1872	do.
5	10°..02′	$10^s.40$	30.34	2530	600 f. m.
5	15°..01′	$15^s.2$	30.20	Lost	...
8	2°..07′	$3^s.75$	30.50	1018	700 f. m.

The additional angle s was computed from the formula $\tan s = \frac{h}{R}$; h varying slightly according to the state of the tide, and R increasing as shown. The gun having been laid by spirit level at 0°, 2°, 4°, etc., the table shows that the additional angles were 16′, 7′, 5′, 4′, etc. No deflections could be satisfactorily determined by the plane tables. The attempt to determine them at 15° failed, owing to the observers at the out plane table being unable to find the splash of the shot.

The gun was trained by means of two carefully made discs, with holes in their centres, which were placed in the bore.

The jump angle was found as described on p. 149, Text-Book of Ordnance and Gunnery. It was $+28'$; this is the result of 9 determinations, the inclination of the piece to its slide varying through about 4°. It was thought that this might sensibly change the jump; but it did not in this case. The jump on the field carriage was, by 4 determinations, found to be $+34'$. In firing for the vertical jump angle, which is that referred to above, it was found that the gun jumped to the left through a horizontal angle of $11'$. Since the gun was held in nearly the same way as when the range firing took place, this may account for the absence of deflection. The ranges were not corrected for the effect of wind.

Resuming the equation

$$\frac{\alpha - a}{R} = b + cR^2,$$

and using the first four tabular entries to determine b and c, we find for the equation connecting angles of elevation and range,

$$\alpha = -28. + .073024R + .00007595R^3.$$

Each observation has here been given equal weight.

In this equation R is in yards and α in minutes of arc. To mark the sight-bar, we have simply to put $R = 100$, 200, etc., and find the corresponding values of α. If $R = 100$, for example, $\alpha = -21'$, by simply shifting the decimal point and adding. Since the jump angle is $+28'$, the angle of projection for 100 yards is $+7'$. Of course, this formula cannot be used for any other gun, nor can it be used outside the limits of the observations on which it depends.

If it were required to determine the range table for a wide variation of angles, it would probably be better to use a succession of such formulæ over separate arcs, rather than to attempt to determine the constants from all the observations. The custom of putting range tables for direct fire by themselves, and those for greater angles elsewhere, is probably conducive to clearness, and has been adopted in some services.

If α is in minutes of arc and θ in circular measure, we have

$$\theta = \frac{\alpha}{60} \cdot \frac{\pi}{180},$$

whence from the equation above,

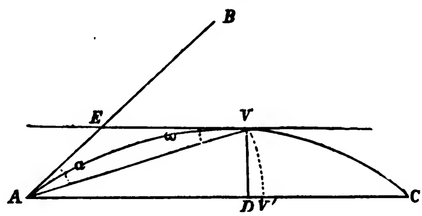
$$\theta = \frac{-8145. + 21.242R + .022095R^3}{10^6},$$

and $$\frac{Rd\theta}{dR} = \frac{21.242R + .04419R^2}{10^6} = \tan\psi,$$

where ψ is the angle of fall at any range R (ψ is the angle between the tangent and the radius vector to the curve, see Rice and Johnson's Diff. Calculus, Art. 317. It is of course assumed in the above expression that the trajectory is rigid, the assumption made being exactly that made in plotting trajectories by polar distortion.) For example, if $R = 1000$, $\psi = 3°44'$.

Thus the equations derived give a quick and easy method of finding the angles of elevation and projection, and angles of fall. The time could similarly be found for all intermediate points; the curve drawn between time of flight and angles of elevation is evidently nearly straight. There is only one assumption made which requires justification; this is that the range or any line is independent of the inclination of the line to the horizontal. For small inclinations, and when the attainable accuracy in all respects is considered, it is evident that we shall come as near the truth by this means as any other now available.

It is always desirable to know the greatest ordinate in the trajectory for any angle of departure; and the following ingenious method of finding it when we have the Range Table, which is taken from Balistique Rationnelle, par J. Baills, is here inserted.



Suppose AVC is the trajectory. Draw a tangent VE at the vertex: then the range corresponding to the angle of projection α and angle of fall ω is AV. Also $\alpha + \omega = BAC$, which is the whole angle of departure for which the trajectory AVC is plotted. Therefore, if we examine the range table, and find an angle of departure α and of fall ω such that their sum shall equal BAC, then the range corresponding is AV, the length of the radius vector drawn from A to the vertex: then

$$AD = AV\cos\omega,$$
$$VD = AV\sin\omega.$$

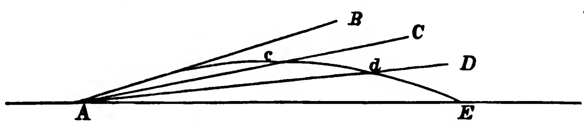
Thus the coordinates of the vertex are known.

It will be noticed in the tabular statement of experimental firing there is an entry of 4 shots at 2°07′ angle of elevation in one place, and of 8 at the same angle in another. Since the barometer and thermometer varied very sensibly at the time of these two records, they are not very suitable to the determination of the mean error of the gun at this range, the object in view in obtaining them. They give, however, a mean error in range of 20 yards at what was, for practical purposes, 1000 yards range. This is a most satisfactory showing for the accuracy of the gun, since the mean errors in range of most guns which are of about similar dimensions is generally as great as 30 yards. The largest single error at this range was 54 yards.

§ III.

It is sometimes convenient for practical purposes to plot graphically the curve which a shot will describe when projected at any given angle. This may evidently be done to any desired degree of accuracy by finding as many points as may be desired by Niven's method. Or, when the angle of departure is small, we may use the process laid down in Example 2, Chapter V.

When the range table of the gun considered is at hand, there is a convenient method which assumes the trajectory to be rigid, which will now be described. Suppose it is required to plot the trajectory for an angle of $n°$. From a horizontal line AE, lay off an



angle $EAB = n°$; then, from AB as origin, lay off BAC, BAD, &c., equal to any convenient unit, say 1°, 2°, &c.; on AC, AD, &c., lay off from A the ranges Ac, Ad, &c., due to angles of departure equal to BAC, BAD, &c.; then $AcdE$ is the trajectory required. For, if we draw a line AC intersecting any trajectory $AcdE$ at the gun and at any point c; then, if the range on AC be independent of the inclination of AC to the horizontal line, the shot will, if projected at an angle BAC from AC, cut AC at c. If now the angles be all multiplied by any convenient factor (usually 5 or 10 in practice), we obtain what is called Plotting Trajectories by Polar Distortion. The method of using these curves is given in Chapter XIV, Text-Book of Ordnance and Gunnery.

BALLISTIC TABLES.

In the use of these tables, the diameter of the projectile is to be taken in inches, and its weight in pounds avoirdupois, the ranges in feet, and the velocities in feet per second. Their use is properly limited to the standard ogival, whose head is struck with a radius of 1½ diameters.

Table I.

Value of K for the Cubic Law of Resistance, Ogival-headed Projectiles (1½ diameter heads).

Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.
f.s.		f.s.		f.s.		f.s.	
100	578.1	780	77.4	1460	100.9	2140	67.5
110	525.5	790	76.8	1470 ...	100.1	2150	67.4
120	481.7	800	76.2	1480	99.4	2160	67.3
130	444.7	810	75.6	1490	98.6	2170	67.2
140	412.9	820	75.2	1500	97.9	2180	67.2
150	385.4	830	75.1	1510	97.1	2190	67.1
160	361.3	840	75.0	1520	96.2	2200	67.0
170	340.1	850	75.0	1530	95.3	2210 ...	66.9
180	321.2	860	75.0	1540	94.4	2220	66.8
190	304.8	870	75.0	1550	93.6	2230	66.8
200	289.0	880	75.0	1560	92.8	2240	66.7
210 ...	275.3	890	75.0	1570	92.0	2250	66.6
220	262.8	900	75.0	1580	91.2	2260	66.5
230	251.3	910	75.0	1590	90.4	2270	66.4
240	240.9	920	75.0	1600	89.7	2280	66.2
250 ...	231.2	930	75.0	1610	89.0	2290	65.9
260	222.4	940	75.0	1620	88.3	2300	65.5
270	214.1	950	75.0	1630	87.6	2310	65.0
280	206.5	960	75.0	1640	86.9	2320	64.4
290	199.8	970	75.0	1650	86.2	2330	63.8
300	192.7	980	75.0	1660	85.5	2340	63.2
310	186.5	990	75.0	1670	84.8	2350	62.6
320	180.8	1000	75.0	1680	84.2	2360 ...	62.0
330	175.5	1010	75.1	1690	83.6	2370 ...	61.4
340	170.6	1020	75.3	1700	83.0	2380	60.8
350	166.0	1030	76.7	1710	82.4	2390	60.2
360	161.9	1040	80.8	1720	81.8	2400	59.6
370	158.0	1050	87.8	1730	81.2	2410	59.0
380	154.4	1060	94.0	1740	80.6	2420	58.4
390	151.1	1070	98.7	1750	80.0	2430	57.8
400	148.0	1080	102.2	1760	79.5	2440	57.2
410	145.2	1090	104.9	1770	78.9	2450	56.7
420	142.5	1100	106.9	1780	78.4	2460	56.2
430	139.8	1110	108.4	1790	77.8	2470	55.7
440	137.2	1120	109.2	1800	77.3	2480	55.2
450	134.6	1130	109.6	1810	76.8	2490	54.8
460	132.0	1140	109.6	1820	76.2	2500	54.4
470	129.4	1150	109.6	1830	75.7	2510	54.0
480	126.9	1160	109.6	1840	75.2	2520	53.7
490	124.4	1170	109.6	1850	74.7	2530	53.4
500	121.9	1180	109.6	1860	74.2	2540	53.1
510	119.6	1190	109.6	1870	73.6	2550	52.9
520	117.8	1200	109.6	1880	73.1	2560	52.7
530	115.0	1210	109.6	1890 ...	72.6	2570	52.6
540	112.8	1220	109.6	1900	72.1	2580	52.5
550	110.7	1230	109.5	1910	71.6	2590	52.5
560	108.7	1240	109.5	1920	71.2	2600	52.4
570	106.7	1250	109.4	1930	70.8	2610	52.4
580	104.6	1260	109.3	1940	70.4	2620	52.4
590	102.5	1270	109.2	1950	70.0	2630	52.3
600	100.5	1280	109.0	1960	69.7	2640	52.3
610	98.6	1290	108.8	1970	69.4	2650	52.3
620	96.8	1300	108.6	1980	69.2	2660	52.2
630	95.1	1310	108.4	1990	69.0	2670	52.2
640	93.5	1320	108.1	2000	68.8	2680	52.2
650	91.9	1330	107.8	2010	68.6	2690	52.1
660	90.5	1340	107.5	2020	68.4	2700	52.1
670	89.1	1350	107.1	2030	68.3	2710	52.1
680	87.7	1360	106.7	2040	68.2	2720	52.0
690	86.3	1370	106.3	2050	68.1	2730	52.0
700	84.9	1380	105.8	2060	68.0	2740	52.0
710	83.7	1390	105.3	2070	67.9	2750 ...	52.0
720	82.6	1400	104.7	2080	67.9	2760	52.0
730	81.6	1410	104.1	2090	67.8	2770	52.0
740	80.6	1420	103.5	2100	67.8	2780	52.0
750	79.6	1430	102.9	2110	67.7	2790	52.0
760	78.7	1440	102.3	2120	67.6	2800	52.0
770	78.0	1450	101.6	2130	67.6		

Mr. Bashforth assumes the weight of a cubic foot of air to be 534.22 grains, which is the weight of a cubic foot of dry air at a temperature of 62° F. under a barometric pressure of 30 inches.

TABLE II.

Distance and velocity table; $\frac{d^2}{w}S = S_{V_1} - S_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
10	1066	1238	1409	1578	1745	1910	2074	2236	2397	2557	166
11	2715	2871	3026	3180	3333	3484	3633	3782	3929	4075	151
12	4220	4363	4506	4647	4787	4926	5064	5200	5336	5471	139
13	5604	5737	5866	5999	6129	6257	6385	6511	6637	6762	129
14	6886	7009	7132	7253	7373	7493	7612	7730	7847	7964	120
15	8079	8194	8309	8422	8535	8647	8758	8868	8978	9087	112
16	9196	9304	9411	9517	9623	9728	9833	9937	10040	10142	105
17	10244	10346	10447	10546	10645	10743	10841	10939	11037	11134	98
18	11230	11326	11421	11516	11610	11704	11797	11890	11982	12074	94
19	12165	12256	12346	12436	12525	12614	12703	12791	12878	12966	89
20	13052	13139	13224	13310	13395	13480	13564	13648	13731	13814	85
21	13896	13979	14060	14142	14223	14303	14384	14463	14543	14622	81
22	14701	14779	14857	14935	15013	15090	·15167	15244	15319	15395	77
23	15470	15545	15620	15694	15768	15842	15916	15989	16061	16134	74
24	16206	16278	16350	16421	16492	16563	16633	16703	16773	16843	71
25	1 6912.1	6981.2	7050.0	7118.5	7186.7	7254.7	7322.4	7389.8	7457.0	7523.9	68.0
26	7590.6	7657.0	7723.2	7789.1	7854.7	7920.1	7985.3	8050.2	8114.8	8179.3	65.4
27	8243.5	8307.5	8371.2	8434.7	8498.0	8561.0	8623.9	8686.4	8748.8	8810.9	63.0
28	1 8872.8	8934.5	8996.0	9057.2	9118.3	9179.1	9239.7	9300.1	9360.3	9420.3	60.8
29	9480.0	9539.6	9598.9	9658.1	9717.0	9775.8	9834.3	9892.6	9950.8	*0008.7	58.7
30	2 0066.5	0124.0	0181.4	0238.5	0295.5	0352.3	0409.0	0465.4	0521.7	0577.7	56.8
31	2 0633.6	0689.3	0744.8	0800.1	0855.3	0910.2	0965.0	1019.6	1074.0	1128.3	55.0
32	1182.4	1236.3	1290.0	1343.5	1396.9	1450.2	1503.2	1556.1	1608.8	1661.4	53.2
33	1713.8	1766.0	1818.1	1870.0	1921.7	1973.3	2024.7	2076.0	2127.1	2178.1	51.6
34	2 2228.9	2279.6	2330.0	2380.4	2430.6	2480.6	2530.5	2580.2	2629.7	2679.1	50.0
35	2728.4	2777.5	2826.4	2875.2	2923.8	2972.3	3020.7	3068.8	3116.9	3164.7	48.5
36	3212.5	3260.1	3307.5	3354.8	3402.0	3449.0	3495.9	3542.6	3589.2	3635.6	47.0
37	2 3682.0	3728.1	3774.2	3820.0	3865.8	3911.4	3956.9	4002.2	4047.4	4092.5	45.6
38	4137.4	4182.2	4226.8	4271.4	4315.7	4360.0	4404.1	4448.1	4491.9	4535.7	44.3
39	4579.2	4622.7	4666.0	4709.2	4752.3	4795.2	4838.1	4880.8	4923.3	4965.7	42.9
40	2 5008.0	5050.2	5092.3	5134.2	5176.0	5217.6	5259.2	5300.6	5341.9	5383.0	41.7
41	5424.0	5464.9	5505.7	5546.4	5586.9	5627.3	5667.6	5707.8	5747.8	5787.8	40.4
42	5827.6	5867.3	5906.9	5946.4	5985.8	6025.0	6064.2	6103.8	6142.2	6181.0	39.3
43	2 6219.8	6258.4	6296.9	6335.3	6373.6	6411.8	6449.9	6487.9	6525.8	6563.6	38.2
44	6601.3	6638.9	6676.4	6713.7	6751.0	6788.2	6825.3	6862.3	6899.3	6936.1	37.2
45	6972.8	7009.4	7046.0	7082.4	7118.8	7155.0	7191.2	7227.3	7263.3	7299.2	36.3
46	2 7335.1	7370.8	7406.5	7442.1	7477.6	7513.0	7548.3	7583.6	7618.8	7653.9	35.4
47	7688.9	7723.8	7758.7	7793.5	7828.2	7862.8	7897.3	7931.8	7966.2	8000.5	34.6
48	8034.7	8068.9	8103.0	8137.0	8170.9	8204.8	8238.6	8272.3	8305.9	8339.5	33.9
49	2 8373.0	8406.5	8439.8	8473.1	8506.4	8539.5	8572.6	8605.6	8638.6	8671.5	33.2
50	8704.3	8737.1	8769.8	8802.4	8835.0	8867.5	8900.0	8932.3	8964.7	8996.9	32.5
51	9029.1	9061.2	9093.2	9125.2	9157.1	9189.0	9220.8	9252.5	9284.2	9315.8	31.9
52	2 9347.3	9378.8	9410.3	9441.6	9472.9	9504.2	9525.4	9566.5	9597.6	9628.7	31.3
53	9659.6	9690.6	9721.4	9752.2	9783.0	9813.7	9844.3	9874.9	9905.4	9935.9	30.7
54	9966.3	9996.7	*0027.0	*0057.3	*0087.5	*0117.7	*0147.8	*0177.8	*0207.8	*0237.8	30.2
55	3 0267.6	0297.5	0327.3	0357.0	0386.7	0416.3	0445.9	0475.4	0504.9	0534.3	29.6
56	0563.6	0592.9	0622.2	0651.4	0680.6	0709.7	0738.7	0767.7	0796.7	0825.6	29.1
57	0854.5	0883.3	0912.1	0940.9	0969.6	0998.2	1026.8	1055.4	1083.9	1112.4	28.6
58	3 1140.8	1169.2	1197.6	1226.0	1254.3	1282.5	1310.8	1339.0	1367.1	1395.2	28.3
59	1423.3	1451.3	1479.3	1507.3	1535.2	1563.0	1590.9	1618.7	1646.4	1674.2	27.9
60	1701.8	1729.5	1757.1	1784.6	1812.2	1839.6	1867.1	1894.5	1921.9	1949.2	27.5
61	3 1976.5	2003.7	2031.0	2058.1	2085.3	2112.4	2139.4	2166.4	2193.4	2220.4	27.1
62	2247.3	2274.2	2301.0	2327.8	2354.5	2381.3	2407.9	2434.6	2461.2	2487.7	26.7
63	2514.3	2540.8	2567.2	2593.6	2620.0	2646.3	2672.6	2698.9	2725.1	2751.3	26.3
64	3 2777.5	2803.6	2829.7	2855.7	2881.7	2907.7	2933.7	2959.6	2985.4	3011.2	26.0
65	3037.0	3062.8	3088.5	3114.2	3139.8	3165.4	3191.0	3216.5	3242.0	3267.4	25.6
66	3292.8	3318.2	3343.5	3368.8	3394.1	3419.3	3444.5	3469.6	3494.7	3519.8	25.2
67	3 3544.8	3569.8	3594.8	3619.8	3644.7	3669.5	3694.3	3719.1	3743.9	3768.6	24.8
68	3793.3	3818.0	3842.6	3867.2	3891.7	3916.2	3940.7	3965.2	3989.6	4014.0	24.5
69	4038.4	4062.7	4087.0	4111.3	4135.6	4159.8	4184.0	4208.1	4232.2	4256.3	24.2

TABLE II.—*Continued.*

Distance and velocity table; $\frac{d^2}{w} S = S_{V_1} - S_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
70	3 4280.4	4304.5	4328.5	4352.4	4376.4	4400.3	4424.1	4448.0	4471.8	4495.5	23.9
71	4519.3	4543.0	4566.6	4590.2	4613.8	4637.4	4660.9	4684.4	4707.8	4731.3	23.5
72	4754.7	4777.9	4801.3	4824.6	4847.9	4871.1	4894.2	4917.4	4940.5	4963.6	23.2
73	3 4986.6	5009.6	5032.6	5055.5	5078.4	5101.3	5124.1	5146.9	5169.6	5192.4	22.8
74	5215.1	5237.7	5260.3	5282.9	5305.5	5328.0	5350.5	5373.0	5395.4	5417.8	22.5
75	5440.2	5462.5	5484.8	5507.1	5529.3	5551.5	5573.7	5595.8	5617.9	5640.0	22.2
76	3 5662.1	5684.1	5706.0	5728.0	5749.9	5771.7	5793.5	5815.3	5837.0	5858.7	21.8
77	5880.4	5902.0	5923.6	5945.1	5966.6	5988.1	6009.5	6030.9	6052.2	6073.6	21.5
78	6094.8	6116.1	6137.3	6158.4	6179.6	6200.7	6221.7	6242.7	6263.7	6284.6	21.1
79	3 6305.5	6326.4	6347.2	6368.0	6388.8	6409.5	6430.2	6450.8	6471.4	6492.0	20.7
80	6512.6	6533.1	6553.6	6574.0	6594.4	6614.8	6635.1	6655.4	6675.7	6695.9	20.4
81	6716.1	6736.3	6756.4	6776.5	6796.5	6816.5	6836.5	6856.4	6876.3	6896.1	20.0
82	3 6916.0	6935.7	6955.5	6975.1	6994.8	7014.4	7033.9	7053.4	7072.9	7092.3	19.6
83	7111.7	7131.0	7150.3	7169.6	7188.8	7207.9	7227.1	7246.1	7265.2	7284.1	19.1
84	7303.1	7322.0	7340.8	7359.6	7378.4	7397.1	7415.8	7434.4	7453.0	7471.5	18.7
85	3 7490.0	7508.5	7526.9	7545.3	7563.6	7581.8	7600.0	7618.2	7636.3	7654.4	18.2
86	7672.4	7690.5	7708.4	7726.4	7744.2	7762.0	7779.9	7797.6	7815.4	7833.0	17.8
87	7850.6	7868.2	7855.8	7903.3	7920.8	7938.2	7955.6	7973.0	7990.3	8007.6	17.4
88	3 8024.8	8042.0	8059.2	8076.3	8093.4	8110.4	8127.4	8144.4	8161.3	8178.2	17.0
89	8195.0	8211.9	8228.6	8245.4	8262.1	8278.7	8295.4	8312.0	8328.5	8345.0	16.6
90	8361.5	8377.9	8394.3	8410.7	8427.0	8443.3	8459.6	8475.8	8492.0	8508.2	16.3
91	3 8524.3	8540.4	8556.4	8572.4	8588.4	8604.3	8620.3	8636.1	8652.0	8667.8	15.9
92	8683.5	8699.3	8715.0	8730.7	8746.3	8761.9	8777.5	8793.0	8808.5	8824.0	15.6
93	8839.4	8854.8	8870.2	8885.5	8900.8	8916.1	8931.3	8946.5	8961.7	8976.8	15.3
94	3 8991.9	9007.0	9022.0	9037.0	9052.0	9066.9	9081.9	9096.7	9111.6	9126.4	15.0
95	9141.2	9156.0	9170.7	9185.4	9200.1	9214.7	9229.3	9243.9	9258.4	9272.9	14.6
96	9287.4	9301.9	9316.3	9330.7	9345.0	9359.4	9373.7	9387.9	9402.2	9416.4	14.3
97	3 9430.6	9444.7	9458.9	9473.0	9487.0	9501.1	9515.1	9529.1	9543.0	9557.0	14.0
98	9570.8	9584.7	9598.6	9612.4	9626.1	9639.9	9653.6	9667.3	9681.0	9604.6	13.7
99	9708.3	9721.9	9735.4	9749.0	9762.5	9775.9	9789.4	9802.8	9816.2	9829.6	13.5
100	3 9842.9	9856.3	9869.6	9882.9	9896.1	9909.3	9922.5	9935.3	9948.8	9961.9	13.2
101	9975.0	9988.1	*0001.1	*0014.1	*0027.1	*0040.0	*0052.9	*0065.8	*0078.7	*0091.5	12.9
102	4 0104.3	0117.1	0129.8	0142.5	0155.2	0167.8	0180.4	0192.9	0205.4	0217.8	12.6
103	4 0230.1	0242.4	0254.6	0266.8	0278.8	0290.8	0302.7	0314.5	0326.2	0337.8	11.9
104	0349.4	0360.8	0372.2	0383.4	0394.5	0405.6	0416.5	0427.3	0438.1	0448.7	11.0
105	0459.2	0469.6	0479.9	0490.0	0500.1	0510.1	0520.0	0529.8	0539.5	0549.2	9.9
106	4 0558.7	0568.2	0577.6	0586.9	0596.2	0605.4	0614.5	0623.6	0632.6	0641.6	9.2
107	0650.5	0659.3	0668.1	0676.9	0685.6	0694.2	0702.8	0711.4	0719.9	0728.4	8.6
108	0736.8	0745.2	0753.6	0761.9	0770.2	0778.4	0786.6	0794.8	0802.9	0811.0	8.2
109	4 0819.0	0827.1	0835.0	0843.0	0850.9	0858.9	0866.7	0874.6	0882.4	0890.2	7.9
110	0897.9	0905.7	0913.4	0921.1	0928.7	0936.4	0944.0	0951.5	0959.1	0966.6	7.6
111	0974.2	0981.6	0989.1	0996.6	1004.0	1011.4	1018.8	1026.2	1033.5	1040.9	7.4
112	4 1048.2	1055.5	1062.8	1070.0	1077.3	1084.5	1091.7	1099.0	1106.1	1113.3	7.2
113	1120.5	1127.6	1134.8	1141.9	1149.0	1156.1	1163.2	1170.2	1177.3	1184.4	7.1
114	1191.4	1198.4	1205.4	1212.4	1219.4	1226.4	1233.3	1240.3	1247.2	1254.1	6.9
115	4 1261.0	1267.9	1274.8	1281.7	1288.6	1295.4	1302.3	1309.1	1315.9	1322.7	6.8
116	1329.5	1336.3	1343.1	1349.8	1356.6	1363.3	1370.0	1376.7	1383.4	1390.1	6.7
117	1396.8	1403.5	1410.1	1416.8	1423.4	1430.0	1436.6	1443.2	1449.8	1456.4	6.6
118	4 1462.9	1469.5	1476.0	1482.6	1489.1	1495.6	1502.1	1508.6	1515.1	1521.5	6.5
119	1528.0	1534.4	1540.9	1547.3	1553.7	1560.1	1566.5	1572.9	1579.2	1585.6	6.4
120	1591.9	1598.3	1604.6	1610.9	1617.2	1623.5	1629.8	1636.1	1642.3	1648.6	6.3
121	4 1654.8	1661.1	1667.3	1673.5	1679.7	1685.9	1692.1	1698.2	1704.4	1710.5	6.2
122	1716.7	1722.8	1728.8	1735.0	1741.1	1747.2	1753.3	1759.4	1765.4	1771.5	6.1
123	1777.5	1783.6	1789.6	1795.6	1801.6	1807.6	1813.6	1819.6	1825.6	1831.5	6.0
124	4 1837.5	1843.4	1849.4	1855.3	1861.2	1867.1	1873.0	1878.9	1884.8	1890.6	5.9
125	1896.5	1902.3	1908.2	1914.0	1919.8	1925.6	1931.5	1937.3	1943.0	1948.8	5.8
126	1954.6	1960.4	1966.1	1971.9	1977.6	1983.3	1989.0	1994.8	2000.5	2006.2	5.7
127	4 2011.8	2017.5	2023.2	2028.9	2034.5	2040.2	2045.8	2051.4	2057.0	2062.7	5.6
128	2068.3	2073.9	2079.5	2085.0	2090.6	2096.2	2101.8	2107.3	2112.9	2118.4	5.6
129	2123.9	2129.4	2135.0	2140.5	2146.0	2151.5	2157.0	2162.4	2167.9	2173.4	5.5

TABLE II.—*Continued.*

Distance and velocity table; $\frac{d^2}{w}\ S = S_{V_1} - S_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
130	4 2178.8	2184.3	2189.7	2195.1	2200.6	2206.0	2211.4	2216.8	2222.2	2227.6	5.4
131	2233.0	2238.4	2243.7	2249.1	2254.5	2259.8	2265.1	2270.5	2275.8	2281.1	5.3
132	2286.4	2291.8	2297.1	2302.4	2307.6	2312.9	2318.2	2323.5	2328.7	2334.0	5.3
133	4 2339.2	2344.5	2349.7	2355.0	2360.2	2365.4	2370.6	2375.8	2381.0	2386.2	5.2
134	2391.4	2396.6	2401.8	2406.9	2412.1	2417.3	2422.4	2427.6	2432.7	2437.8	5.2
135	2443.0	2448.1	2453.2	2458.3	2463.4	2468.5	2473.6	2478.7	2483.8	2488.9	5.1
136	4 2493.9	2499.0	2504.1	2509.1	2514.2	2519.2	2524.3	2529.3	2534.3	2539.4	5.0
137	2544.4	2549.4	2554.4	2559.4	2564.4	2569.4	2574.4	2579.4	2584.3	2589.3	5.0
138	2594.3	2599.2	2604.2	2609.1	2614.1	2619.0	2624.0	2628.9	2633.8	2638.8	4.9
139	4 2643.7	2648.6	2653.5	2658.4	2663.3	2668.2	2673.1	2678.0	2682.9	2687.8	4.9
140	2692.6	2697.5	2702.4	2707.2	2712.1	2717.0	2721.8	2726.7	2731.5	2736.3	4.9
141	2741.2	2746.0	2750.8	2755.7	2760.5	2765.3	2770.1	2774.9	2779.7	2784.5	4.8
142	4 2789.3	2794.1	2798.9	2803.7	2808.5	2813.2	2818.0	2822.8	2827.5	2832.3	4.8
143	2837.1	2841.8	2846.6	2851.3	2856.0	2860.8	2865.5	2870.2	2875.0	2879.7	4.7
144	2884.4	2889.1	2893.8	2898.6	2903.3	2908.0	2912.7	2917.4	2922.1	2926.7	4.7
145	4 2931.4	2936.1	2940.8	2945.5	2950.1	2954.8	2959.5	2964.1	2968.8	2973.5	4.7
146	2978.1	2982.8	2987.4	2992.1	2996.7	3001.3	3006.0	3010.6	3015.2	3019.9	4.6
147	3024.5	3029.1	3033.7	3038.4	3043.0	3047.6	3052.2	3056.8	3061.4	3066.0	4.6
148	4 3070.6	3075.2	3079.8	3084.4	3089.0	3093.5	3098.1	3102.7	3107.3	3111.8	4.6
149	3116.4	3121.0	3125.6	3130.1	3134.7	3139.2	3143.8	3148.3	3152.9	3157.4	4.6
150	3162.0	3166.5	3171.0	3175.6	3180.1	3184.6	3189.2	3193.7	3198.2	3202.7	4.5
151	4 3207.2	3211.8	3216.3	3220.8	3225.3	3229.8	3234.3	3238.8	3243.3	3247.8	4 5
152	3252.3	3256.8	3261.3	3265.8	3270.3	3274.8	3279.3	3283.8	3288.3	3292.8	4.5
153	3297.2	3301.7	3306.2	3310.6	3315.1	3319.6	3324.1	3328.5	3333.0	3337.5	4.5
154	4 3342.0	3346.4	3350.9	3355.3	3359.8	3364.3	3368.7	3373.2	3377.6	3382.1	4.5
155	3386.5	3391.0	3395.4	3399.9	3404.3	3408.7	3413.2	3417.6	3422.0	3426.5	4.4
156	3430.9	3435.3	3439.8	3444.2	3448.6	3453.0	3457.4	3461.9	3466.3	3470.7	4.4
157	4 3475.1	3479.5	3483.9	3488.3	3492.7	3497.1	3501.5	3505.9	3510.3	3514.7	4.4
158	3519.1	3523.5	3527.9	3532.3	3536.7	3541.1	3545.4	3549.8	3554.2	3558.6	4.4
159	3563.0	3567.3	3571.7	3576.1	3580.4	3584.8	3589.1	3593.5	3597.9	3602.2	4.4
160	4 3606.6	3610.9	3615.3	3619.6	3624.0	3628.3	3632.6	3637.0	3641.3	3645.7	4.3
161	3650.0	3654.3	3658.7	3663.0	3667.3	3671.6	3676.0	3680.3	3684.6	3688.9	4.3
162	3693.3	3697.6	3701.9	3706.1	3710.5	3714.8	3719.1	3723.4	3727.7	3732.0	4.3
163	4 3736.3	3740.6	3744.9	3749.2	3753.5	3757.8	3762.1	3766.4	3770.6	3774.9	4.3
164	3779.2	3783.5	3787.8	3792.0	3796.3	3800.6	3804.9	3809.1	3813.4	3817.6	4.3
165	3281.9	3826.2	3830.4	3834.7	3838.9	3843.2	3847.4	3851.7	3855.9	3860.2	4.3
166	4 3864.4	3868.7	3872.9	3877.2	3881.4	3885.6	3889.9	3894.1	3898.8	3902.5	4.2
167	3906.8	3911.0	3915.2	3919.5	3923.7	3927.9	3932.1	3936.3	3940.5	3944.7	4.2
168	3949.0	3953.2	3957.4	3961.6	3965.8	3970.0	3974.2	3978.4	3982.6	3986.7	4.2
169	4 3990.9	3995.1	3999.3	4003.5	4007.7	4011.9	4016.0	4020.2	4024.4	4028.6	4.2
170	4032.7	4036.9	4041.1	4045.2	4049.4	4053.6	4057.7	4061.9	4066.0	4070.2	4.2
171	4074.3	4078.5	4082.6	4086.8	4090.9	4095.1	4099.2	4103.3	4107.5	4111.6	4.1
172	4 4115.7	4119.9	4124.0	4128.1	4132.3	4136.4	4140.5	4144.6	4148.7	4152.9	4.1
173	4157.0	4161.1	4165.2	4169.3	4173.4	4177.5	4181.6	4185.7	4189.8	4193.9	4.1
174	4198.0	4202.1	4206.2	4210.3	4214.4	4218.5	4222.6	4226.7	4230.8	4234.8	4.1
175	4 4238.9	4243.0	4247.1	4251.2	4255.3	4259.3	4263.4	4267.5	4271.5	4275.6	4.1
176	4279.6	4283.7	4287.8	4291.8	4295.9	4300.0	4304.0	4308.8	4312.1	4316.1	4.1
177	4320.2	4324.2	4328.3	4332.3	4336.4	4340.4	4344.4	4348.5	4352.5	4356.5	4.0
178	4 4360.5	4364.6	4368.6	4372.6	4376.6	4380.7	4384.7	4388.7	4392.7	4396.7	4.0
179	4400.7	4404.7	4408.8	4412.8	4416.8	4420.8	4424.8	4428.8	4432.8	4436.8	4.0
180	4440.8	4444.7	4448.7	4452.7	4456.7	4460.7	4464.7	4468.7	4472.6	4476.6	4.0
181	4 4480.6	4484.6	4488.5	4492.5	4496.5	4500.5	4504.4	4508.4	4512.4	4516.3	4.0
182	4520.3	4524.2	4528.2	4532.2	4536.1	4540.1	4544.0	4548.0	4551.9	4555.9	4.0
183	4559.8	4563.7	4567.7	4571.6	4575.6	4579.5	4583.4	4587.4	4591.3	4595.2	3.9
184	4 4599.2	4603.1	4607.0	4610.9	4614.9	4618.8	4622.7	4626.6	4630.5	4634.4	3.9
185	4638.4	4642.3	4646.2	4650.1	4654.0	4657.9	4661.8	4665.7	4669.6	4673.5	3.9
186	4677.4	4681.3	4685.2	4689.1	4693.0	4696.9	4700.8	4704.6	4708.5	4712.4	3.9
187	4 4716.3	4720.2	4724.1	4727.9	4731.8	4735.7	4739.6	4743.4	4747.3	4751.2	3.9
188	4755.0	4758.9	4762.8	4766.7	4770.5	4774.4	4778.2	4782.1	4786.0	4789.8	3.9
189	4793.7	4797.5	4801.4	4805.2	4809.1	4812.9	4816.8	4820.6	4824.5	4828.3	3.8

Table II.—*Continued.*

Distance and velocity table; $\frac{d^2}{w}S = S_{V_1} - S_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
190	4 4832.2	4836.0	4439.8	4843.7	4847.5	4851.4	4855.2	4859.0	4862.8	4866.7	3.8
191	4870.5	4874.3	4878.1	4882.0	4885.8	4889.6	4893.4	4897.3	4901.1	4904.9	3.8
192	4908.7	4912.5	4916.3	4920.1	4923.9	4927.7	4931.5	4935.3	4939.1	4942.9	3.8
193	4 4946.7	4950.5	4954.3	4958.1	4961.9	4965.7	4969.4	4973.2	4977.0	4980.7	3.8
194	4984.5	4988.3	4992.1	4995.8	4999.6	5003.4	5007.1	5010.9	5014.7	5018.4	3.8
195	5022.2	5025.9	5029.7	5033.4	5037.2	5040.9	5044.7	5048.4	5052.1	5055.9	3.7
196	4 5059.6	5063.4	5067.1	5070.8	5074.6	5078.3	5082.0	5085.7	5089.4	5093.1	3.7
197	5096.9	5100.6	5104.3	5108.0	5111.7	5115.4	5119.1	5122.8	5126.5	5130.2	3.7
198	5133.9	5137.5	5141.2	5144.9	5148.6	5152.3	5156.0	5159.6	5163.3	5166.9	3.7
199	4 5170.6	5174.3	5177.9	5181.6	5185.2	5188.9	5192.5	5196.2	5199.8	5203.4	3.6
200	5207.1	5210.7	5214.3	5218.0	5221.6	5225.2	5228.8	5232.5	5236.1	5239.7	3.6
201	5243.3	5246.9	5250.5	5254.1	5257.7	5261.3	5264.9	5268.5	5272.1	5275.7	3.6
202	4 5279.2	5282.8	5286.4	5290.0	5293.6	5297.2	5300.7	5304.3	5307.8	5311.4	3.6
203	5314.9	5318.5	5322.0	5325.6	5329.1	5332.7	5336.2	5339.7	5343.3	5346.8	3.5
204	5350.3	5353.8	5357.3	5360.9	5364.4	5367.9	5371.4	5374.9	5378.4	5391.9	3.5
205	4 5385.4	5388.9	5392.4	5395.9	5399.4	5402.9	5406.3	5409.8	5413.3	5416.7	3.5
206	5420.2	5423.7	5427.1	5430.6	5434.1	5437.5	5441.0	5444.4	5447.8	5451.3	3.5
207	5454.7	5458.1	5461.6	5465.0	5468.4	5471.9	5475.3	5478.7	5482.1	5485.5	3.4
208	4 5488.9	5492.3	5495.7	5499.1	5502.5	5505.9	5509.3	5512.7	5516.1	5519.4	3.4
209	5522.8	5526.2	5529.6	5532.9	5536.3	5539.7	5543.0	5546.4	5549.7	5553.1	3.4
210	5556.4	5559.8	5563.1	5566.4	5569.8	5573.1	5576.5	5579.8	5583.1	5586.4	3.3
211	4 5589.7	5593.0	5596.4	5599.7	5603.0	5606.3	5609.6	5612.9	5616.2	5619.5	3.3
212	5622.8	5626.1	5629.3	5632.6	5635.9	5639.2	5642.5	5645.7	5649.0	5652.3	3.3
213	5655.5	5658.8	5662.0	5665.3	5668.6	5671.8	5675.1	5678.3	5681.5	5684.8	3.2
214	4 5688.0	5691.2	5694.5	5697.7	5700.9	5704.2	5707.4	5710.6	5713.8	5717.0	3.2
215	5720.2	5723.4	5726.6	5729.9	5733.1	5736.3	5739.5	5742.6	5745.8	5749.0	3.2
216	5752.2	5755.4	5758.6	5761.8	5764.9	5768.1	5771.3	5774.4	5777.6	5780.8	3.2
217	4 5783.9	5787.1	5790.2	5793.4	5796.6	5799.7	5802.9	5806.0	5809.1	5812.2	3.1
218	5815.4	5818.5	5821.6	5824.8	5827.9	5831.0	5834.1	5837.3	5840.4	5843.5	3.1
219	5846.6	5849.7	5852.8	5855.9	5859.0	5862.1	5865.2	5868.3	5871.4	5874.4	3.1
220	4 5877.5	5880.6	5883.7	5886.8	5889.9	5893.0	5896.0	5899.1	5902.1	5905.2	3.1
221	5908.3	5911.3	5914.4	5917.4	5920.5	5923.6	5926.6	5929.6	5932.7	5935.7	3.0
222	5938.7	5941.8	5944.8	5947.8	5950.9	5953.9	5956.9	5959.9	5963.0	5966.0	3.0
223	4 5969.0	5972.0	5975.0	5978.0	5981.0	5984.0	5987.0	5990.0	5993.0	5996.0	3.0
224	5999.0	6002.0	6004.9	6007.9	6010.9	6013.9	6016.9	6019.8	6022.8	6025.8	3.0
225	6028.7	6031.7	6034.6	6037.6	6040.5	6043.5	6046.5	6049.4	6052.4	6055.3	3.0
226	4 6058.3	6061.2	6064.1	6067.1	6070.0	6072.9	6075.9	6078.8	6081.7	6084.7	2.9
227	6087.6	6090.5	6093.4	6096.3	6099.3	6102.2	6105.1	6108.0	6110.9	6113.8	2.9
228	6116.7	6119.6	6122.5	6125.4	6128.3	6131.2	6134.1	6137.0	6139.9	6142.8	2.9
229	4 6145.7	6148.6	6151.5	6154.4	6157.3	6160.1	6163.1	6166.0	6168.8	6171.7	2.9
230	6174.6	6177.5	6180.4	6183.3	6186.2	6189.4	6191.9	6194.8	6197.7	6200.6	2.9
231	6203.5	6206.4	6209.3	6212.1	6215.0	6217.9	6220.8	6223.7	6226.6	6229.5	2.9
232	4 6232.3	6235.2	6238.1	6241.0	6243.9	6246.8	6249.7	6252.6	6255.4	6258.3	2.9
233	6261.2	6264.1	6267.0	6269.9	6272.8	6275.7	6278.6	6281.5	6284.3	6287.2	2.9
234	6290.1	6293.0	6295.9	6298.8	6301.7	6304.6	6307.5	6310.4	6313.3	6316.2	2.9
235	4 6319.0	6322.0	6324.9	6327.7	6330.6	6333.5	6336.4	6339.3	6342.2	6345.1	2.9
236	6348.0	6350.9	6353.8	6356.7	6359.6	6362.5	6365.4	6368.3	6371.2	6374.1	2.9
237	6377.0	6379.9	6382.8	6385.7	6388.6	6391.5	6394.4	6397.3	6400.2	6403.1	2.9
238	4 6406.0	6408.9	6411.8	6414.8	6417.7	6420.6	6423.5	6426.4	6429.3	6432.2	2.9
239	6435.1	6438.0	6440.9	6443.8	6446.8	6449.7	6452.6	6455.5	6458.4	6461.3	2.9
240	6464.2	6467.1	6470.1	6473.0	6475.9	6478.8	6481.7	6484.6	6487.6	6490.5	2.9
241	4 6493.4	6496.3	6499.2	6502.2	6505.1	6508.0	6510.9	6513.8	6516.8	6519.7	2.9
242	6522.6	6525.6	6528.5	6531.4	6534.3	6537.3	6540.2	6543.1	6546.1	6549.0	2.9
243	6551.9	6554.9	6557.8	6560.7	6563.7	6566.6	6569.5	6572.5	6575.4	6578.3	2.9
244	4 6581.3	6584.2	6587.2	6590.1	6593.0	6596.0	6598.9	6601.8	6604.8	6607.7	2.9
245	6610.6	6613.6	6616.5	6619.5	6622.4	6625.3	6628.3	6631.2	6634.2	6637.1	2.9
246	6640.1	6643.0	6645.9	6748.9	6651.8	6654.8	6657.7	6660.6	6663.6	6666.5	2.9
247	4 6669.5	6672.4	6675.4	6678.3	6681.3	6684.2	6687.2	6690.1	6693.0	6696.0	2.9
248	6698.9	6701.9	6704.8	6707.8	6710.7	6713.7	6716.6	6719.6	6722.5	6725.5	2.9
249	6728.4	6731.3	6734.3	6737.2	6740.2	6743.1	6746.1	6749.0	6752.0	6754.9	2.9

TABLE II.—*Continued.*

Distance and velocity table ; $\frac{d^2}{w} S = S_{V_1} - S_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
250	4 6757.8	6760.7	6763.7	6766.7	6769.6	6772.6	6775.5	6778.4	6781.4	6784.3	2.9
251	6787.3	6790.2	6793.1	6796.1	6799.0	6802.0	6804.9	6807.8	6810.8	6813.7	2.9
252	6816.6	6819.6	6822.5	6825.4	6828.4	6831.3	6834.2	6837.1	6840.1	6843.0	2.9
253	4 6845.9	6848.8	6851.8	6854.7	6857.6	6860.5	6863.5	6866.4	6869.3	6872.2	2.9
254	6875.1	6878.1	6881.0	6883.9	6886.8	6889.7	6892.6	6895.6	6898.5	6901.4	2.9
255	6904.3	6907.2	6910.1	6913.0	6915.9	6918.8	6921.7	6924.6	6927.5	6930.4	2.9
256	4 6933.3	6936.2	6939.1	6942.0	6944.9	6947.8	6950.6	6953.5	6956.4	6959.3	2.9
257	6962.2	6965.0	6967.9	6970.8	6973.7	6976.5	6979.4	6982.3	6985.1	6988.0	2.9
258	6990.9	6993.7	6996.6	6999.4	7002.3	7005.1	7008.0	7010.8	7013.7	7016.5	2.9
259	4 7019.4	7022.2	7025.0	7027.9	7030.7	7033.5	7036.4	7039.2	7042.0	7044.8	2.8
260	7047.7	7050.5	7053.3	7056.1	7058.9	7061.7	7064.5	7067.4	7070.2	7073.0	2.8
261	7075.8	7078.6	7081.4	7084.2	7087.0	7089.7	7092.5	7095.3	7098.1	7100.9	2.8
262	4 7103.7	7106.5	7109.2	7112.0	7114.8	7117.6	7120.3	7123.1	7125.9	7128.6	2.8
263	7131.4	7134.2	7136.9	7139.7	7142.4	7145.2	7147.9	7150.7	7153.4	7156.2	2.8
264	7158.9	7161.7	7164.4	7167.1	7169.9	7172.6	7175.4	7178.1	7180.8	7183.5	2.7
265	4 7186.3	7189.0	7191.7	7194.4	7197.1	7199.9	7202.6	7205.3	7208.0	7210.7	2.7
266	7213.4	7216.1	7218.8	7221.5	7224.2	7226.9	7229.6	7232.3	7235.0	7237.7	2.7
267	7240.4	7243.1	7245.8	7248.5	7251.2	7253.8	7256.5	7259.2	7261.9	7264.5	2.7
268	4 7267.2	7269.9	7272.5	7275.2	7277.9	7280.5	7283.2	7285.9	7288.5	7291.2	2.7
269	7293.8	7296.5	7299.1	7301.8	7304.4	7307.1	7309.7	7312.3	7315.0	7317.6	2.6
270	7320.2	7322.9	7325.5	7328.1	7330.8	7333.4	7336.0	7338.6	7341.2	7343.9	2.6
271	4 7346.5	7349.1	7351.7	7354.3	7356.9	7359.5	7362.1	7364.7	7367.3	7369.9	2.6
272	7372.5	7375.1	7377.7	7380.3	7382.9	7385.5	7388.1	7390.7	7393.3	7395.8	2.6
273	7398.4	7401.0	7403.6	7406.2	7408.7	7411.3	7413.9	7416.4	7419.0	7421.6	2.6
274	4 7424.1	7426.7	7429.3	7431.8	7434.4	7436.9	7439.5	7442.0	7444.6	7447.1	2.6
275	7449.7	7452.2	7454.8	7457.3	7459.8	7462.4	7464.9	7467.4	7470.0	7472.5	2.5
276	7475.0	7477.5	7480.1	7482.6	7485.1	7487.6	7490.1	7492.7	7495.2	7497.7	2.5
277	4 7500.2	7502.7	7505.2	7507.7	7510.2	7512.7	7515.2	7517.7	7520.2	7522.7	2.5
278	7525.2	7527.7	7530.1	7532.6	7535.1	7537.6	7540.1	7542.6	7545.0	7547.5	2.5
279	7550.0	7552.4	7554.9	7557.4	7559.9	7562.3	7564.8	7567.2	7569.7	7572.2	2.5
280	4 7574.6	7577.1	7579.5	7582.0	7584.4	7586.8	7589.3	7591.7	7594.2	7596.6	2.4
281	7599.0	7601.5	7603.9	7606.4	7608.8	7611.2	7613.6	7616.1	7618.5	7620.9	2.4
282	7623.3	7625.7	7628.2	7630.6	7633.0	7635.4	7637.8	7640.2	7642.6	7645.0	2.4
283	4 7647.4	7649.8	7652.2	7654.6	7657.0	7659.4	7661.8	7664.2	7666.6	7669.0	2.4
284	7671.3	7673.7	7676.1	7678.5	7680.9	7683.3	7685.6	7688.0	7690.4	7692.7	2.4
285	7695.1	7697.5	7699.8	7702.2	7704.6	7706.9	7709.3	7711.6	7714.0	7716.4	2.4
286	4 7718.7	7721.1	7723.4	7725.8	7728.1	7730.4	7732.8	7735.1	7737.5	7739.8	2.3
287	7742.1	7744.5	7746.8	7749.1	7751.5	7753.8	7756.1	7758.4	7760.8	7763.1	2.3
288	7765.4	7767.7	7770.0	7772.4	7774.7	7777.0	7779.3	7781.6	7783.9	7786.2	2.3
289	4 7788.5	7790.8	7793.1	7795.4	7797.7	7800.0	7802.3	7804.6	7806.9	7809.2	2.3
290	7811.5										

TABLE III.

Time and velocity table; $\frac{d^2}{w} T = T_{V_1} - T_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
10	75.399	77.111	78.790	80.437	82.052	83.636	85.190	86.715	88.212	89.682	1.584
11	91.125	92.542	93.934	95.301	96.644	97.964	99.261	*00.536	*01.789	*03.021	1.320
12	1 04.232	05.423	06.595	07.748	08.883	09.999	11.097	12.178	13.243	14.291	1.116
13	1 15.323	16.339	17.340	18.326	19.297	20.254	21.196	22.124	23.039	23.941	.957
14	24.830	25.706	26.570	27.422	28.262	29.091	29.908	30.714	31.509	32.294	.829
15	33.068	33.832	34.586	35.331	36.066	36.792	37.508	38.215	38.913	39.602	.726
16	1 40.283	40.955	41.618	42.273	42.920	43.559	44.190	44.813	45.429	46.038	.639
17	46.640	47.235	47.823	48.404	48.978	49 546	50.107	50.662	51.211	51.754	.563
18	52.291	52.822	53.347	53.867	54.381	54.890	55.393	55.890	56.382	56.869	.509
19	1 57.351	57.828	58.300	58.767	59.229	59.686	60.138	60.586	61.029	61.468	.457
20	61.902	62.332	62.758	63.180	63.598	64.012	64.422	64.828	65.230	65.628	.414
21	66.022	66.412	66.798	67.181	67.560	67.936	68.308	68.676	69.041	69.403	.376
22	1 69.762	70.118	70.470	70.819	71.165	71.508	71.848	72.185	72.519	72.850	.343
23	73.179	73.505	73.828	74.148	74.465	74.780	75.092	75.401	75.708	76.012	.315
24	76.314	76.613	76.909	77.203	77.494	77.783	78.070	78.354	78.636	78.916	.289
25	1 79.194	79.470	79.743	80.014	80.283	80.550	80.815	81.078	81.339	81.598	.267
26	81.855	82.110	82.363	82.614	82.863	83.110	83.355	83.598	83.839	84.079	.247
27	84.317	84.553	84.787	85.020	85.251	85.481	85.709	85.935	86.160	86.382	.230
28	1 86.604	86.824	87.042	87.259	87.474	87.688	87.900	88.111	88.320	88.528	.214
29	88.734	88.939	89.143	89.345	89.546	89.745	89.943	90.140	90.335	90.529	.199
30	90.721	90.912	91.102	91.291	91.478	91.664	91.849	92.033	92.216	92.397	.186
31	1 92.577	92.756	92.934	93.111	93.287	93.461	93.634	93.806	93.971	94.147	.174
32	94.316	94.484	94.651	94.817	94.982	95.146	95.309	95.471	95.632	95.792	.164
33	95.951	96.109	96.266	96.422	96.577	96.731	96.884	97.036	97.187	97.338	.154
34	1 97.488	97.637	97.785	97.932	98.078	98.223	98.367	98.510	98.652	98.794	.145
35	98.935	99.075	99.214	99.352	99.490	99.627	99.763	99.898	*00.032	*00.166	.137
36	2 00.299	00.431	00.562	00.692	00.822	00.951	01.079	01.206	01.333	01.459	.129
37	2 01.585	01.710	01.834	01.957	02.080	02.202	02.323	02.443	02.563	02.682	.122
38	02.801	02.919	03.036	03.152	03.268	03.383	03.497	03.610	03.723	03.835	.115
39	03.947	04.058	04.168	04.278	04.387	04.496	04.604	04.711	04.818	04.924	.109
40	20 5.0299	5.1349	5.2393	5.3432	5.4466	5.5494	5.6517	5.7534	5.8546	5.9553	.1028
41	6.0554	6.1550	6.2540	6.3525	6.4505	6.5480	6.6450	6.7414	6.8373	6.9327	.0975
42	7.0276	7.1220	7.2159	7.3093	7.4022	7.4947	7.5867	7.6782	7.7693	7.8599	.0925
43	20 7.9501	8.0398	8.1291	8.2179	8.3063	8.3942	8.4817	8.5687	8.6553	8.7415	.0879
44	8.8272	8.9125	8.9974	9.0819	9.1660	9.2497	9.3330	9.4159	9.4984	9.5805	.0837
45	9.6622	9.7435	9.8244	9.9050	9.9852	*0.0651	*0.1446	*0.2237	*0.3025	*0.3809	.0799
46	21 0.4590	0.5367	0.6140	0.6910	0.7677	0.8440	0.9200	0.9956	1.0709	1.1459	.0763
47	1.2205	1.2948	1.3687	1.4423	1.5156	1.5886	1.6613	1.7336	1.8056	1.8773	.0730
48	1.9487	2.0198	2.0906	2.1611	2.2313	2.3012	2.3708	2.4401	2.5091	2.5779	.0699
49	21 2.6464	2.7146	2.7825	2.8501	2.9174	2.9845	3.0513	3.1178	3.1841	3.2501	.0671
50	3.3159	3.3814	3.4466	3.5116	3.5763	3.6408	3.7050	3.7689	3.8326	3.8960	.0645
51	3.9592	4.0221	4.0848	4.1472	4.2094	4.2713	4.3330	4.3944	4.4556	4.5165	.0619
52	21 4.5772	4.6377	4.6979	4.7579	4.8177	4.8773	4.9367	4.9958	5.0547	5.1134	.0596
53	5.1719	5.2302	5.2882	5.3460	5.4036	5.4610	5.5182	5.5752	5.6320	5.6886	.0574
54	5.7450	5.8012	5.8572	5.9130	5.9686	6.0240	6.0792	6.1342	6.1890	6.2436	.0554
55	21 6.2980	6.3522	6.4062	6.4600	6.5136	6.5670	6.6202	6.6732	6.7260	6.7786	.0534
56	6.8311	6.8834	6.9355	6.9874	7.0391	7.0907	7.1421	7.1933	7.2444	7.2953	.0516
57	7.3460	7.3965	7.4469	7.4971	7.5471	7.5970	7.6467	7.6962	7.7456	7.7948	.0499
58	21 7.8438	7.8928	7.9417	7.9904	8.0389	8.0873	8.1356	8.1837	8.2316	8.2793	.0483
59	8.3271	8.3746	8.4220	8.4692	8.5163	8.5632	8.6100	8.6566	8.7031	8.7494	.0468
60	8.7957	8.8417	8.8877	8.9334	8.9791	9.0246	9.0700	9.1152	9.1603	9.2052	.0454
61	21 9.2501	9.2947	9.3393	9.3837	9.4280	9.4721	9.5161	9.5600	9.6037	9.6473	.0441
62	9.6908	9.7341	9.7773	9.8204	9.8633	9.9063	9.9489	9.9914	*0.0338	*0.0761	.0428
63	22 0.1183	0.1604	0.2023	0.2441	0.2858	0.3273	0.3687	0.4100	0.4512	0.4922	.0415
64	22 0.5333	0.5740	0.6147	0.6552	0.6957	0.7360	0.7762	0.8163	0.8563	0.8962	.0403
65	0.9359	0.9755	1.0151	1.0544	1.0937	1.1328	1.1718	1.2107	1.2495	1.2881	.0391
66	1.3267	1.3651	1.4034	1.4416	1.4797	1.5177	1.5555	1.5933	1.6309	1.6684	.0379
67	22 1.7059	1.7432	1.7804	1.8175	1.8545	1.8914	1.9281	1.9648	2.0014	2.0378	.0368
68	2.0742	2.1105	2.1466	2.1827	2.2186	2.2545	2.2902	2.3259	2.3614	2.3969	.0358
69	2.4322	2.4675	2.5027	2.5377	2.5727	2.6076	2.6424	2.6771	2.7117	2.7462	.0348

TABLE III.—*Continued.*

Time and velocity table; $\frac{d^2}{w}T = T_{V_1} - T_{V_2}$.

Ogival-headed projectiles, $1\frac{1}{2}$ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
70	22 2.7806	2.8150	2.8492	2.8833	2.9174	2.9513	2.9852	3.0189	3.0526	3.0862	.0339
71	3.1196	3.1530	3.1863	3.2195	3.2526	3.2856	3.3185	3.3513	3.3840	3.4167	.0330
72	3.4492	3.4816	3.5140	3.5462	3.5784	3.6105	3.6424	3.6743	3.7061	3.7378	.0320
73	22 3.7694	3.8009	3.8323	3.8636	3.8949	3.9260	3.9571	3.9881	4.0189	4.0497	.0311
74	4.0804	4.1110	4.1416	4.1720	4.2024	4.2326	4.2628	4.2929	4.3230	4.3529	.0302
75	4.3828	4.4125	4.4422	4.4719	4.5014	4.5308	4.5602	4.5895	4.6187	4.6478	.0294
76	22 4.6769	4.7058	4.7347	4.7635	4.7922	4.8208	4.8493	4.8777	4.9060	4.9348	.0286
77	4.9624	4.9905	5.0185	5.0464	5.0742	5.1020	5.1296	5.1572	5.1847	5.2121	.0277
78	5.2394	5.2666	5.2937	5.3208	5.3478	5.3747	5.4015	5.4282	5.4549	5.4814	.0268
79	22 5.5079	5.5343	5.5606	5.5869	5.6130	5.6391	5.6652	5.6911	5.7170	5.7428	.0261
80	5.7685	5.7941	5.8197	5.8452	5.8706	5.8959	5.9212	5.9463	5.9714	5.9965	.0253
81	6.0214	6.0463	6.0711	6.0959	6.1205	6.1451	6.1696	6.1941	6.2184	6.2427	.0245
82	22 6.2669	6.2910	6.3151	6.3390	6.3629	6.3867	6.4104	6.4340	6.4576	6.4810	.0237
83	6.5044	6.5277	6.5509	6.5740	6.5971	6.6201	6.6430	6.6658	6.6885	6.7111	.0229
84	6.7337	6.7562	6.7786	6.8009	6.8232	6.8454	6.8675	6.8895	6.9114	6.9333	.0221
85	22 6.9551	6.9768	6.9984	7.0200	7.0415	7.0629	7.0842	7.1055	7.1267	7.1478	.0214
86	7.1688	7.1898	7.2107	7.2315	7.2522	7.2729	7.2935	7.3140	7.3345	7.3549	.0206
87	7.3752	7.3954	7.4156	7.4357	7.4558	7.4757	7.4956	7.5155	7.5353	7.5550	.0199
88	22 7.5746	7.5942	7.6137	7.6332	7.6526	7.6719	7.6912	7.7104	7.7295	7.7486	.0193
89	7.7677	7.7866	7.8055	7.8244	7.8431	7.8618	7.8805	7.8991	7.9176	7.9360	.0187
90	7.9544	7.9727	7.9909	8.0091	8.0272	8.0452	8.0632	8.0812	8.0990	8.1168	.0180
91	22 8.1346	8.1523	8.1699	8.1875	8.2050	8.2225	8.2399	8.2573	8.2746	8.2918	.0174
92	8.3090	8.3261	8.3432	8.3602	8.3772	8.3941	8.4109	8.4277	8.4445	8.4611	.0169
93	8.4778	8.4943	8.5109	8.5273	8.5437	8.5601	8.5764	8.5927	8.6089	8.6250	.0163
94	22 8.6411	8.6572	8.6732	8.6892	8.7051	8.7209	8.7367	8.7525	8.7682	8.7838	.0158
95	8.7994	8.8150	8.8305	8.8459	8.8613	8.8767	8.8920	8.9073	8.9225	8.9376	.0153
96	8.9528	8.9678	8.9828	8.9978	9.0128	9.0276	9.0425	9.0573	9.0720	9.0867	.0149
97	22 9.1014	9.1160	9.1306	9.1451	9.1595	9.1740	9.1884	9.2027	9.2170	9.2312	.0144
98	9.2454	9.2596	9.2737	9.2878	9.3018	9.3158	9.3298	9.3437	9.3575	9.3713	.0140
99	9.3851	9.3989	9.4126	9.4262	9.4398	9.4534	9.4670	9.4805	9.4939	9.5073	.0136
100	22 9.5207	9.5340	9.5473	9.5606	9.5738	9.5869	9.6001	9.6132	9.6262	9.6392	.0132
101	9.6522	9.6651	9.6780	9.6908	9.7036	9.7164	9.7291	9.7418	9.7544	9.7670	.0127
102	9.7796	9.7921	9.8046	9.8170	9.8294	9.8417	9.8540	9.8662	9.8783	9.8904	.0123
103	22 9.9024	9.9144	9.9262	9.9380	9.9496	9.9612	9.9727	9.9841	9.9954	*0.0066	.0115
104	23 0.0177	0.0287	0.0396	0.0504	0.0610	0.0716	0.0820	0.0923	0.1025	0.1126	.0105
105	0.1226	0.1325	0.1423	0.1520	0.1615	0.1710	0.1804	0.1897	0.1988	0.2079	.0094
106	23 0.2170	0.2259	0.2347	0.2435	0.2522	0.2609	0.2694	0.2780	0.2864	0.2948	.0086
107	0.3031	0.3114	0.3196	0.3278	0.3359	0.3439	0.3520	0.3599	0.3678	0.3757	.0080
108	0.3835	0.3913	0.3990	0.4067	0.4143	0.4219	0.4295	0.4370	0.4445	0.4519	.0076
109	23 0.4593	0.4667	0.4740	0.4813	0.4885	0.4958	0.5030	0.5101	0.5172	0.5243	.0072
110	0.5314	0.5384	0.5454	0.5524	0.5593	0.5662	0.5731	0.5800	0.5868	0.5936	.0069
111	0.6004	0.6071	0.6139	0.6206	0.6272	0.6339	0.6405	0.6471	0.6537	0.6603	.0066
112	23 0.6668	0.6733	0.6798	0.6863	0.6928	0.6992	0.7056	0.7120	0.7184	0.7248	.0064
113	0.7311	0.7374	0.7437	0.7500	0.7563	0.7625	0.7688	0.7750	0.7812	0.7874	.0062
114	0.7936	0.7997	0.8059	0.8120	0.8181	0.8242	0.8303	0.8364	0.8424	0.8484	.0061
115	23 0.8545	0.8605	0.8665	0.8726	0.8787	0.8847	0.8906	0.8965	0.9024	0.9083	.0059
116	0.9142	0.9200	0.9259	0.9317	0.9375	0.9433	0.9490	0.9548	0.9605	0.9663	.0058
117	0.9720	0.9777	0.9833	0.9890	0.9947	1.0003	1.0059	1.0115	1.0171	1.0227	.0056
118	23 1.0283	1.0338	1.0394	1.0449	1.0504	1.0559	1.0614	1.0669	1.0723	1.0778	.0055
119	1.0832	1.0886	1.0940	1.0994	1.1048	1.1101	1.1154	1.1208	1.1261	1.1314	.0054
120	1.1367	1.1420	1.1473	1.1525	1.1578	1.1630	1.1682	1.1734	1.1786	1.1838	.0052
121	23 1.1889	1.1941	1.1992	1.2043	1.2095	1.2146	1.2196	1.2247	1.2298	1.2348	.0051
122	1.2399	1.2449	1.2499	1.2549	1.2599	1.2649	1.2698	1.2748	1.2797	1.2847	.0050
123	1.2896	1.2945	1.2994	1.3043	1.3091	1.3140	1.3188	1.3237	1.3285	1.3333	.0049
124	23 1.3381	1.3429	1.3477	1.3524	1.3572	1.3619	1.3667	1.3714	1.3761	1.3808	.0047
125	1.3855	1.3902	1.3948	1.3995	1.4041	1.4088	1.4134	1.4180	1.4226	1.4272	.0046
126	1.4318	1.4364	1.4410	1.4455	1.4501	1.4546	1.4591	1.4636	1.4681	1.4726	.0045
127	23 1.4771	1.4816	1.4860	1.4905	1.4949	1.4993	1.5038	1.5082	1.5126	1.5170	.0044
128	1.5214	1.5257	1.5301	1.5345	1.5388	1.5431	1.5475	1.5518	1.5561	1.5604	.0043
129	1.5647	1.5690	1.5732	1.5775	1.5818	1.5860	1.5902	1.5945	1.5987	1.6029	.0042

TABLE III.—*Continued.*

Time and velocity table; $\frac{d^2}{w} T = T_{V_1} - T_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	sec.	+
130	23 1.6071	1.6113	1.6155	1.6196	1.6238	1.6280	1.6321	1.6362	1.6404	1.6445	.0042
131	1.6486	1.6527	1.6568	1.6609	1.6650	1.6690	1.6731	1.6772	1.6812	1.6852	.0041
132	1.6893	1.6933	1.6973	1.7013	1.7053	1.7093	1.7133	1.7173	1.7212	1.7252	.0040
133	23 1.7291	1.7331	1.7370	1.7410	1.7449	1.7488	1.7527	1.7566	1.7605	1.7644	.0039
134	1.7682	1.7721	1.7760	1.7798	1.7837	1.7875	1.7913	1.7952	1.7990	1.8028	.0038
135	1.8066	1.8104	1.8142	1.8179	1.8217	1.8255	1.8292	1.8330	1.8367	1.8405	.0038
136	23 1.8442	1.8479	1.8517	1.8554	1.8591	1.8628	1.8665	1.8702	1.8738	1.8775	.0037
137	1.8812	1.8848	1.8885	1.8921	1.8958	1.8994	1.9030	1.9067	1.9103	1.9139	.0036
138	1.9175	1.9211	1.9247	1.9282	1.9318	1.9354	1.9390	1.9425	1.9461	1.9496	.0036
139	23 1.9532	1.9567	1.9602	1.9638	1.9673	1.9708	1.9743	1.9778	1.9813	1.9848	.0035
140	1.9883	1.9918	1.9952	1.9987	2.0022	2.0056	2.0091	2.0125	2.0160	2.0194	.0035
141	2.0228	2.0263	2.0297	2.0331	2.0365	2.0399	2.0433	2.0467	2.0501	2.0535	.0034
142	23 2.0569	2.0602	2.0636	2.0670	2.0703	2.0737	2.0770	2.0804	2.0837	2.0870	.0034
143	2.0904	2.0937	2.0970	2.1003	2.1036	2.1069	2.1102	2.1135	2.1168	2.1201	.0033
144	2.1234	2.1267	2.1299	2.1332	2.1364	2.1397	2.1430	2.1462	2.1494	2.1527	.0033
145	23 2.1559	2.1591	2.1624	2.1656	2.1688	2.1720	2.1752	2.1784	2.1816	2.1848	.0032
146	2.1880	2.1912	2.1944	2.1975	2.2007	2.2039	2.2071	2.2102	2.2134	2.2165	.0032
147	2.2197	2.2228	2.2260	2.2291	2.2322	2.2354	2.2385	2.2416	2.2447	2.2478	.0031
148	23 2.2509	2.2540	2.2571	2.2602	2.2633	2.2664	2.2695	2.2726	2.2757	2.2787	.0031
149	2.2818	2.2849	2.2879	2.2910	2.2940	2.2971	2.3001	2.3032	2.3062	2.3093	.0030
150	2.3123	2.3153	2.3183	2.3214	2.3244	2.3274	2.3304	2.3334	2.3364	2.3394	.0030
151	23 2.3424	2.3454	2.3484	2.3514	2.3543	2.3573	2.3603	2.3633	2.3662	2.3692	.0030
152	2.3722	2.3751	2.3781	2.3810	2.3840	2.3869	2.3899	2.3928	2.3958	2.3987	.0029
153	2.4016	2.4046	2.4075	2.4104	2.4133	2.4162	2.4192	2.4221	2.4250	2.4279	.0029
154	23 2.4308	2.4337	2.4366	2.4395	2.4424	2.4453	2.4481	2.4510	2.4539	2.4568	.0029
155	2.4597	2.4625	2.4654	2.4683	2.4711	2.4740	2.4768	2.4797	2.4825	2.4854	.0029
156	2.4882	2.4911	2.4939	2.4967	2.4996	2.5024	2.5052	2.5080	2.5108	2.5137	.0028
157	23 2.5165	2.5193	2.5221	2.5249	2.5277	2.5305	2.5333	2.5361	2.5389	2.5416	.0028
158	2.5444	2.5472	2.5500	2.5528	2.5555	2.5583	2.5611	2.5638	2.5666	2.5693	.0028
159	2.5721	2.5748	2.5776	2.5803	2.5831	2.5858	2.5885	2.5913	2.5940	2.5967	.0027
160	23 2.5994	2.6022	2.6049	2.6076	2.6103	2.6130	2.6157	2.6184	2.6211	2.6238	.0027
161	2.6265	2.6292	2.6319	2.6346	2.6373	2.6400	2.6426	2.6453	2.6480	2.6506	.0027
162	2.6533	2.6560	2.6586	2.6613	2.6640	2.6666	2.6693	2.6719	2.6745	2.6772	.0026
163	23 2.6798	2.6825	2.6851	2.6877	2.6903	2.6930	2.6956	2.6982	2.7008	2.7034	.0026
164	2.7061	2.7087	2.7113	2.7139	2.7165	2.7191	2.7217	2.7243	2.7268	2.7294	.0026
165	2.7320	2.7346	2.7372	2.7398	2.7423	2.7449	2.7475	2.7500	2.7526	2.7552	.0026
166	23 2.7577	2.7603	2.7628	2.7654	2.7679	2.7705	2.7730	2.7756	2.7781	2.7806	.0025
167	2.7832	2.7857	2.7882	2.7908	2.7933	2.7958	2.7983	2.8008	2.8034	2.8059	.0025
168	2.8084	2.8109	2.8134	2.8159	2.8184	2.8209	2.8234	2.8258	2.8283	2.8308	.0025
169	23 2.8333	2.8358	2.8383	2.8407	2.8432	2.8457	2.8481	2.8506	2.8531	2.8555	.0025
170	2.8580	2.8604	2.8629	2.8653	2.8678	2.8702	2.8726	2.8751	2.8775	2.8799	.0024
171	2.8824	2.8848	2.8872	2.8896	2.8921	2.8945	2.8969	2.8993	2.9017	2.9041	.0024
172	23 2.9065	2.9089	2.9113	2.9137	2.9161	2.9185	2.9209	2.9233	2.9257	2.9281	.0024
173	2.9304	2.9328	2.9352	2.9376	2.9399	2.9423	2.9447	2.9470	2.9494	2.9518	.0024
174	2.9541	2.9565	2.9588	2.9612	2.9635	2.9659	2.9682	2.9705	2.9729	2.9752	.0023
175	23 2.9776	2.9799	2.9822	2.9845	2.9869	2.9892	2.9915	2.9938	2.9961	2.9985	.0023
176	3.0008	3.0031	3.0054	3.0077	3.0100	3.0123	3.0146	3.0169	3.0192	3.0215	.0023
177	3.0237	3.0260	3.0283	3.0306	3.0329	3.0351	3.0374	3.0397	3.0420	3.0442	.0023
178	23 3.0465	3.0488	3.0510	3.0533	3.0555	3.0578	3.0600	3.0623	3.0645	3.0668	.0023
179	3.0690	3.0713	3.0735	3.0757	3.0780	3.0802	3.0824	3.0847	3.0869	3.0891	.0022
180	3.0913	3.0935	3.0958	3.0980	3.1002	3.1024	3.1045	3.1068	3.1090	3.1112	.0022
181	23 3.1134	3.1156	3.1178	3.1200	3.1222	3.1244	3.1266	3.1287	3.1309	3.1331	.0022
182	3.1353	3.1375	3.1396	3.1418	3.1440	3.1461	3.1483	3.1505	3.1526	3.1548	.0022
183	3.1569	3.1591	3.1613	3.1634	3.1656	3.1677	3.1698	3.1720	3.1741	3.1763	.0021
184	23 3.1784	3.1805	3.1827	3.1848	3.1869	3.1891	3.1912	3.1933	3.1954	3.1975	.0021
185	3.1997	3.2018	3.2039	3.2060	3.2081	3.2102	3.2123	3.2144	3.2165	3.2186	.0021
186	3.2207	3.2228	3.2249	3.2270	3.2291	3.2312	3.2333	3.2353	3.2374	3.2395	.0021
187	23 3.2416	3.2437	3.2457	3.2478	3.2499	3.2520	3.2540	3.2561	3.2582	3.2602	.0021
188	3.2623	3.2643	3.2664	3.2685	3.2705	3.2726	3.2746	3.2767	3.2787	3.2808	.0021
189	3.2828	3.2848	3.2869	3.2889	3.2909	3.2930	3.2950	3.2970	3.2991	3.3011	.0020

Table III.—*Continued.*

Time and velocity table; $\frac{d^2}{w} T = T_{V_1} - T_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
190	23 3.3031	3.3051	3.3072	3.3092	3.3112	3.3132	3.3152	3.3172	3.3192	3.3212	.0020
191	3.3233	3.3253	3.3273	3.3293	3.3313	3.3333	3.3353	3.3372	3.3392	3.3412	.0020
192	3.3432	3.3452	3.3472	3.3492	3.3511	3.3531	3.3551	3.3571	3.3590	3.3610	.0020
193	23 3.3630	3.3649	3.3669	3.3689	3.3708	3.3728	3.3747	3.3767	3.3786	3.3806	.0020
194	3.3825	3.3845	3.3864	3.3884	3.3903	3.3922	3.3942	3.3961	3.3980	3.4000	.0019
195	3.4019	3.4038	3.4057	3.4077	3.4096	3.4115	3.4134	3.4153	3.4172	3.4192	.0019
196	23 3.4211	3.4230	3.4249	3.4268	3.4287	3.4306	3.4325	3.4344	3.4362	3.4381	.0019
197	3.4400	3.4419	3.4438	3.4457	3.4476	3.4494	3.4513	3.4532	3.4550	3.4569	.0019
198	3.4588	3.4606	3.4625	3.4644	3.4662	3.4681	3.4699	3.4718	3.4736	3.4755	.0019
199	23 3.4773	3.4791	3.4810	3.4828	3.4846	3.4865	3.4883	3.4901	3.4920	3.4938	.0018
200	3.4956	3.4974	3.4992	3.5010	3.5028	3.5047	3.5065	3.5083	3.5101	3.5119	.0018
201	3.5137	3.5155	3.5172	3.5190	3.5208	3.5226	3.5244	3.5262	3.5280	3.5297	.0018
202	23 3.5315	3.5333	3.5351	3.5368	3.5386	3.5404	3.5421	3.5439	3.5456	3.5474	.0018
203	3.5492	3.5509	3.5527	3.5544	3.5561	3.5579	3.5596	3.5614	3.5631	3.5648	.0017
204	3.5666	3.5683	3.5700	3.5717	3.5735	3.5752	3.5769	3.5786	3.5803	3.5820	.0017
205	23 3.5837	3.5854	3.5871	3.5888	3.5905	3.5922	3.5939	3.5956	3.5973	3.5990	.0017
206	3.6007	3.6024	3.6040	3.6057	3.6074	3.6091	3.6107	3.6124	3.6141	3.6157	.0017
207	3.6174	3.6191	3.6207	3.6224	3.6240	3.6257	3.6273	3.6290	3.6306	3.6323	.0016
208	23 3.6339	3.6355	3.6372	3.6388	3.6404	3.6420	3.6437	3.6453	3.6469	3.6485	.0016
209	3.6502	3.6518	3.6534	3.6550	3.6566	3.6582	3.6598	3.6614	3.6630	3.6646	.0016
210	3.6662	3.6678	3.6694	3.6710	3.6726	3.6741	3.6757	3.6773	3.6789	3.6805	.0016
211	23 3.6820	3.6836	3.6852	3.6867	3.6883	3.6899	3.6914	3.6930	3.6946	3.6961	.0016
212	3.6977	3.6992	3.7008	3.7023	3.7039	3.7054	3.7070	3.7085	3.7100	3.7116	.0015
213	3.7131	3.7146	3.7162	3.7177	3.7192	3.7207	3.7223	3.7238	3.7253	3.7268	.0015
214	23 3.7283	3.7298	3.7313	3.7329	3.7344	3.7359	3.7374	3.7389	3.7404	3.7419	.0015
215	3.7434	3.7448	3.7463	3.7478	3.7493	3.7508	3.7523	3.7538	3.7552	3.7567	.0015
216	3.7582	3.7597	3.7612	3.7626	3.7641	3.7656	3.7670	3.7685	3.7700	3.7714	.0015
217	23 3.7729	3.7743	3.7758	3.7772	3.7787	3.7801	3.7816	3.7830	3.7845	3.7859	.0014
218	3.7874	3.7888	3.7902	3.7917	3.7931	3.7945	3.7960	3.7974	3.7988	3.8002	.0014
219	3.8016	3.8031	3.8045	3.8059	3.8073	3.8087	3.8101	3.8115	3.8129	3.8144	.0114
220	23 3.8158	3.8172	3.8186	3.8200	3.8214	3.8227	3.8241	3.8255	3.8269	3.8283	.0014
221	3.8297	3.8311	3.8325	3.8338	3.8352	3.8366	3.8380	3.8394	3.8407	3.8421	.0014
222	3.8435	3.8448	3.8462	3.8476	3.8489	3.8503	3.8517	3.8530	3.8544	3.8557	.0014
223	23 3.8571	3.8584	3.8598	3.8611	3.8625	3.8638	3.8651	3.8665	3.8678	3.8692	.0013
224	3.8705	3.8718	3.8732	3.8745	3.8758	3.8772	3.8785	3.8798	3.8811	3.8824	.0013
225	3.8838	3.8851	3.8864	3.8877	3.8890	3.8903	3.8916	3.8930	3.8943	3.8956	.0013
226	23 3.8969	3.8982	3.8995	3.9008	3.9021	3.9034	3.9047	3.9059	3.9072	3.9085	.0013
227	3.9098	3.9111	3.9124	3.9137	3.9150	3.9162	3.9175	3.9188	3.9201	3.9214	.0013
228	3.9226	3.9239	3.9252	3.9264	3.9277	3 9290	3.9303	3.9315	3.9328	3.9341	.0013
229	23 3.9353	3.9366	3.9378	3.9391	3.9404	3.9416	3.9429	3.9441	3.9454	3.9467	.0013
230	3.9479	3.9492	3.9504	3.9517	3.9529	3.9542	3.9554	3.9567	3.9579	3.9592	.0013
231	3.9604	3.9617	3.9629	3.9642	3.9654	3.9667	3.9679	3.9692	3.9704	3.9716	.0012
232	23 3.9729	3.9741	3.9754	3.9766	3.9779	3.9791	3.9803	3.9816	3.9828	3.9841	.0012
233	3.9853	3.9866	3.9878	3.9890	3.9903	3.9915	3.9927	3.9940	3.9952	3.9965	.0012
234	3.9977	3.9989	4.0002	4.0014	4.0026	4.0039	4.0051	4.0063	4.0076	4.0088	.0012
235	23 4.0100	4.0113	4.0125	4.0137	4.0150	4.0162	4.0174	4.0186	4.0199	4.0211	.0012
236	4.0223	4.0236	4.0248	4.0260	4.0272	4.0284	4.0297	4.0309	4.0321	4.0334	.0012
237	4.0346	4.0358	4.0370	4.0383	4.0395	4.0407	4.0419	4.0431	4.0444	4.0456	.0012
238	23 4.0468	4.0480	4.0492	4.0505	4.0517	4.0529	4.0541	4.0553	4.0566	4.0578	.0012
239	4.0590	4.0602	4.0614	4.0626	4.0639	4.0651	4.0663	4.0675	4.0687	4.0699	.0012
240	4.0711	4.0724	4.0736	4.0748	4.0760	4.0772	4.0784	4.0796	4.0809	4.0821	.0012
241	23 4.0833	4.0845	4.0857	4.0869	4.0881	4.0893	4.0905	4.0917	4.0930	4.0942	.0012
242	4.0954	4.0966	4.0978	4.0990	4.1002	4.1014	4.1026	4.1038	4.1050	4.1062	.0012
243	4.1074	4.1087	4.1099	4.1111	4.1123	4.1135	4.1147	4.1159	4.1171	4.1183	.0012
244	23 4.1195	4.1207	4.1219	4.1231	4.1243	4.1255	4.1267	4.1279	4.1291	4.1303	.0012
245	4.1315	4.1327	4.1339	4.1351	4.1363	4.1375	4.1387	4.1399	4.1411	4.1423	.0012
246	4.1435	4.1447	4.1459	4.1471	4.1483	4.1495	4.1506	4.1518	4.1530	4.1542	.0012
247	23 4.1554	4.1566	4.1578	4.1590	4.1602	4.1614	4.1626	4.1638	4.1649	4.1661	.0012
248	4.1673	4.1685	4.1697	4.1709	4.1721	4.1733	4.1744	4.1756	4.1768	4.1780	.0012
249	4.1792	4.1804	4.1815	4.1827	4.1839	4.1851	4.1863	4.1874	4.1886	4.1898	.0012

Table III.—*Continued.*

Time and velocity table ; $\frac{d^2}{w} T = T_{V_1} - T_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
250	23 4.1910	4.1922	4.1933	4.1945	4.1957	4.1969	4.1980	4.1992	4.2004	4.2015	.0012
251	4.2027	4.2039	4.2051	4.2062	4.2074	4.2086	4.2097	4.2109	4.2121	4.2132	.0012
252	4.2144	4.2156	4.2167	4.2179	4.2190	4.2202	4.2214	4.2225	4.2237	4.2248	.0012
253	23 4.2260	4.2272	4.2283	4.2295	4.2306	4.2318	4.2329	4.2341	4.2352	4.2364	.0012
254	4.2375	4.2387	4.2398	4.2410	4.2421	4.2433	4.2444	4.2455	4.2467	4.2478	.0011
255	4.2490	4.2501	4.2513	4.2524	4.2535	4.2547	4.2558	4.2569	4.2581	4.2592	.0011
256	23 4.2603	4.2615	4.2626	4.2637	4.2648	4.2660	4.2671	4.2682	4.2693	4.2705	.0011
257	4.2716	4.2727	4.2738	4.2749	4.2760	4.2772	4.2783	4.2794	4.2805	4.2816	.0011
258	4.2827	4.2838	4.2849	4.2860	4.2871	4.2682	4.2893	4.2904	4.2915	4.2926	.0011
259	23 4.2937	4.2948	4.2959	4.2970	4.2981	4.2992	4.3003	4.3014	4.3025	4.3036	.0011
260	4.3046	4.3057	4.3068	4.3079	4.3090	4.3101	4.3111	4.3122	4.3133	4.3144	.0011
261	·4.3154	4.3165	4.3176	4.3187	4.3197	4.3208	4.3219	4.3229	4.3240	4.3250	.0011
262	23 4.3261	4.3272	4.3282	4.3293	4.3303	4.3314	4.3325	4.3335	4.3346	4.3356	.0011
263	4.3367	4.3377	4.3388	4.3398	4.3409	4.3419	4.3429	4.3440	4.3450	4.3461	.0010
264	4.3471	4.3482	4.3492	4.3502	4.3513	4.3523	4.3533	4.3544	4.3554	4.3564	.0010
265	23 4.3574	4.3585	4.3595	4.3605	4.3615	4.3626	4.3636	4.3646	4.3656	4.3667	.0010
266	4.3677	4.3687	4.3697	4.3707	4.3717	4.3728	4.3738	4.3748	4.3758	4.3768	.0010
267	4.3778	4.3788	4.3798	4.3808	4.3818	4.3828	4.3838	4.3848	4.3858	4.3868	.0010
268	23 4.3878	4.3888	4.3898	4.3908	4.3918	4.3928	4.3938	4.3948	4.3958	4.3968	.0010
269	4.3977	4.3987	4.3997	4.4007	4.4017	4.4027	4.4036	4.4046	4.4056	4.4066	.0010
270	4.4075	4.4085	4.4095	4.4105	4.4114	4.4124	4.4134	4.4143	4.4153	4.4163	.0010
271	23 4.4172	4.4182	4.4192	4.4201	4.4211	4.4220	4.4230	4.4240	4.4249	4.4259	.0010
272	4.4268	4.4278	4.4287	4.4297	4.4307	4.4316	4.4326	4.4335	4.4344	4.4354	.0010
273	4.4363	4.4373	4.4382	4.4392	4.4401	4.4411	4.4420	4.4429	4.4439	4.4448	.0009
274	23 4.4457	4.4467	4.4476	4.4485	4.4495	4.4504	4.4513	4.4523	4.4532	4.4541	.0009
275	4.4551	4.4560	4.4569	4.4578	4.4587	4.4597	4.4606	4.4615	4.4624	4.4633	.0009
276	4.4643	4.4652	4.4661	4.4670	4.4679	4.4688	4.4697	4.4706	4.4715	4.4725	.0009
277	23 4.4734	4.4743	4.4752	4.4761	4.4770	4.4779	4.4788	4.4797	4.4806	4.4815	.0009
278	4.4824	4.4833	4.4842	4.4850	4.4859	4.4868	4.4877	4.4886	4.4895	4.4904	.0009
279	4.4913	4.4922	4.4930	4.4939	4.4948	4.4957	4.4966	4.4975	4.4983	4.4992	.0009
280	23 4.5001	4.5010	4.5018	4.5027	4.5036	4.5045	4.5053	4.5062	4.5071	4.5080	.0009
281	4.5088	4.5097	4.5105	4.5114	4.5123	4.5131	4.5140	4.5148	4.5157	4.5166	.0009
282	4.5174	4.5183	4.5191	4.5200	4.5208	4.5217	4.5226	4.5234	4.5243	4.5251	.0009
283	23 4.5260	4.5268	4.5277	4.5285	4.5293	4.5302	4.5310	4.5319	4.5327	4.5336	.0008
284	4.5344	4.5352	4.5361	4.5369	4.5378	4.5386	4.5394	4.5403	4.5411	4.5419	.0008
285	4.5427	4.5436	4.5444	4.5452	4.5461	4.5469	4.5477	4.5485	4.5494	4.5502	.0008
286	23 4.5510	4.5518	4.5527	4.5535	4.5543	4.5551	4.5559	4.5567	4.5576	4.5584	.0008
287	4.5592	4.5600	4.5608	4.5616	4.5624	4.5632	4.5641	4.5648	4.5657	4.5665	.0008
288	4.5673	4.5681	4.5689	4.5697	4.5705	4.5713	4.5721	4.5729	4.5737	4.5745	.0008
289	23 4.5753	4.5761	4.5769	4.5777	4.5785	4.5793	4.5800	4.5808	4.5816	4.5824	.0008
290	4.5832										

TABLE IV.

Inclination and velocity table; $\frac{d^2}{w} D = D_{V_1} - D_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
40	0	.4838	.9640	1.4407	1.9137	2.3830	2.8488	3.3110	3.7689	4.2240
41	4.6757	5.1240	5.5688	6.0101	6.4482	6.8828	7.3141	7.7421	8.1660	8.5874
42	9.0056	9.4207	9.8327	10.2410	10.6467	11.0496	11.4494	11.8462	12.2397	12.6306
43	13.0187	13.4039	13.7862	14.1652	14.5419	14.9159	15.2872	15.6557	16.0211	16.3843
44	16.7450	17.1030	17.4585	17.8110	18.1614	18.5094	18.8549	19.1980	19.5383	19.8766
45	20.2125	20.5460	20.8772	21.2054	21.5320	21.8565	22.1788	22.4989	22.8169	23.1327
46	23.4463	23.7578	24.0671	24.3736	24.6788	24.9821	25.2834	25.5927	25.8801	26.1756
47	26.4691	26.7607	27.0503	27.3376	27.6234	27.9075	28.1897	28.4702	28.7486	29.0254
48	29.3006	29.5739	29.8455	30.1151	30.3833	30.6498	30.9147	31.1779	31.4393	31.6993
49	31.9576	32.2143	32.4695	32.7227	32.9747	33.2253	33.4743	33.7219	33.9679	34.2125
50	34.4557	34.6973	34.9375	35.1761	35.4134	35.6493	35.8837	36.1167	36.3480	36.5783
51	36.8073	37.0349	37.2613	37.4862	37.7099	37.9323	38.1534	38.3731	38.5914	38.8086
52	39.0246	39.2394	39.4529	39.6651	39.8762	40.0860	40.2947	40.5022	40.7083	40.9135
53	41.1175	41.3204	41.5221	41.7225	41.9221	42.1205	42.3179	42.5142	42.7095	42.9037
54	43.0967	43.2887	43.4795	43.6690	43.8578	44.0456	44.2324	44.4182	44.6031	44.7870
55	44.9698	45.1516	45.3325	45.5122	45.6910	45.8689	46.0457	46.2217	46.3964	46.5705
56	46.7437	46.9160	47.0874	47.2581	47.4277	47.5965	47.7644	47.9314	48.0973	48.2625
57	48.4270	48.5906	48.7534	48.9153	49.0764	49.2368	49.3963	49.5551	49.7130	49.8701
58	50.0265	50.1822	50.3370	50.4909	50.6442	50.7968	50.9487	51.0999	51.2505	51.4002
59	51.5492	51.6975	51.8451	51.9917	52.1378	52.2832	52.4280	52.5721	52.7155	52.8583
60	53.0003	53.1417	53.2825	53.4224	53.5618	53.7005	53.8386	53.9761	54.1130	54.2492
61	54.3847	54.5196	54.6539	54.7875	54.9205	55.0529	55.1846	55.3158	55.4462	55.5761
62	55.7054	55.8342	55.9623	56.0899	56.2169	56.3433	56.4690	56.5942	56.7188	56.8428
63	56.9663	57.0891	57.2114	57.3330	57.4542	57.5749	57.6950	57.8146	57.9338	58.0523
64	58.1703	58.2878	58.4046	58.5209	58.6367	58.7521	58.8669	58.9832	59.0949	59.2081
65	59.3209	59.4332	59.5449	59.6562	59.7669	59.8772	59.9869	60.0961	60.2047	60.3130
66	60.4207	60.5280	60.6348	60.7411	60.8470	60.9523	61.0572	61.1616	61.2654	61.3688
67	61.4719	61.5744	61.6766	61.7783	61.8796	61.9804	62.0808	62.1807	62.2802	62.3793
68	62.4779	62.5761	62.6739	62.7711	62.8680	62.9646	63.0607	63.1565	63.2519	63.3468
69	63.4414	63.5356	63.6294	63.7227	63.8157	63.9084	64.0006	64.0924	64.1838	64.2749
70	64.3656	64.4559	64.5459	64.6356	64.7249	64.8137	64.9022	64.9903	65.0779	65.1652
71	65.2522	65.3388	65.4250	65.5107	65.5962	65.6813	65.7660	65.8504	65.9345	66.0182
72	66.1015	66.1845	66.2671	66.3494	66.4313	66.5128	66.5940	66.6749	66.7553	66.8355
73	66.9153	66.9949	67.0740	67.1529	67.2314	67.3096	67.3875	67.4649	67.5422	67.6190
74	67.6955	67.7717	67.8476	67.9231	67.9983	68.0733	68.1479	68.2223	68.2964	68.3702
75	68.4436	68.5168	68.5896	68.6620	68.7342	68.8062	68.8778	68.9492	69.0204	69.0912
76	69.1617	69.2318	69.3017	69.3712	69.4404	69.5094	69.5780	69.6464	69.7145	69.7823
77	69.8497	69.9169	69.9838	70.0503	70.1166	70.1826	70.2483	70.3137	70.3787	70.4436
78	70.5082	70.5725	70.6365	70.7004	70.7639	70.8271	70.8901	70.9527	71.0149	71.0770
79	71.1388	71.2004	71.2617	71.3228	71.3837	71.4442	71.5045	71.5646	71.6244	71.6839
80	71.7432	71.8023	71.8611	71.9196	71.9779	72.0359	72.0937	72.1513	72.2086	72.2656
81	72.3225	72.3791	72.4354	72.4915	72.5473	72.6030	72.6584	72.7135	72.7685	72.8232
82	72.8776	72.9317	72.9856	73.0393	73.0927	73.1458	73.1988	73.2514	73.3038	73.3560
83	73.4079	73.4596	73.5111	73.5622	73.6132	73.6639	73.7145	73.7648	73.8149	73.8647
84	73.9143	73.9636	74.0127	74.0615	74.1101	74.1585	74.2067	74.2546	74.3023	74.3498
85	74.3971	74.4441	74.4910	74.5376	74.5839	74.6301	74.6760	74.7217	74.7670	74.8123
86	74.8573	74.9022	74.9468	74.9912	75.0355	75.0795	75.1233	75.1669	75.2104	75.2536
87	75.2966	75.3395	75.3821	75.4246	75.4668	75.5089	75.5507	75.5924	75.6339	75.6752
88	75.7163	75.7572	75.7980	75.8385	75.8788	75.9190	75.9590	75.9988	76.0384	76.0778
89	76.1171	76.1562	76.1952	76.2339	76.2725	76.3109	76.3492	76.3873	76.4252	76.4629
90	76.5005	76.5379	76.5751	76.6121	76.6490	76.6857	76.7223	76.7588	76.7951	76.8312
91	76.8671	76.9029	76.9385	76.9739	77.0092	77.0444	77.0794	77.1142	77.1489	77.1835
92	77.2179	77.2522	77.2863	77.3203	77.3541	77.3878	77.4213	77.4547	77.4879	77.5210
93	77.5540	77.5868	77.6195	77.6520	77.6844	77.7167	77.7488	77.7807	77.8125	77.8442
94	77.8757	77.9071	77.9384	77.9695	78.0005	78.0314	78.0622	78.0929	78.1234	78.1538
95	78.1841	78.2142	78.2442	78.2741	78.3039	78.3335	78.3630	78.3924	78.4216	78.4508
96	78.4798	78.5087	78.5375	78.5622	78.5947	78.6231	78.6514	78.6796	78.7076	78.7356
97	78.7634	78.7911	78.8188	78.8463	78.8736	78.9009	78.9280	78.9551	78.9819	79.0087
98	79.0354	79.0621	79.0886	79.1150	79.1413	79.1675	79.1936	79.2195	79.2454	79.2712
99	79.2968	79.3224	79.3478	79.3731	79.3983	79.4234	79.4484	79.4734	79.4982	79.5230

Table IV.—*Continued.*

Inclination and velocity table; $\frac{d^2}{w} D = D_{V_1} - D_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
100	79.5476	79.5722	79.5966	79.6210	79.6453	79.6695	79.6935	79.7175	79.7414	79.7652
101	79.7889	79.8124	79.8359	79.8593	79.8826	79.9058	79.9289	79.9519	79.9748	79.9976
102	80.0203	80.0430	80.0655	80.0879	80.1102	80.1324	80.1544	80.1763	80.1981	80.2197
103	80.2412	80.2625	80.2837	80.3048	80.3256	80.3462	80.3667	80.3869	80.4071	80.4270
104	80.4466	80.4661	80.4854	80.5045	80.5234	80.5420	80.5605	80.5787	80.5967	80.6145
105	80.6321	80.6495	80.6667	80.6835	80.7003	80.7169	80.7333	80.7495	80.7654	80.7813
106	80.7970	80.8126	80.8280	80.8432	80.8583	80.8733	80.8882	80.9029	80.9175	80.9319
107	80.9463	80.9606	80.9747	80.9886	81.0026	81.0164	81.0301	81.0437	81.0573	81.0707
108	81.0841	81.0973	81.1105	81.1236	81.1366	81.1495	81.1624	81.1751	81.1877	81.2003
109	81.2129	81.2253	81.2377	81.2501	81.2623	81.2745	81.2866	81.2986	81.3105	81.3224
110	81.3342	81.3460	81.3578	81.3695	81.3811	81.3927	81.4042	81.4156	81.4269	81.4382
111	81.4495	81.4607	81.4719	81.4829	81.4939	81.5049	81.5159	81.5268	81.5377	81.5485
112	81.5593	81.5700	81.5807	81.5913	81.6019	81.6124	81.6230	81.6334	81.6439	81.6543
113	81.6647	81.6750	81.6853	81.6955	81.7057	81.7159	81.7260	81.7361	81.7462	81.7562
114	81.7662	81.7761	81.7861	81.7960	81.8058	81.8156	81.8254	81.8351	81.8448	81.8545
115	81.8641	81.8737	81.8833	81.8929	81.9024	81.9119	81.9213	81.9307	81.9401	81.9495
116	81.9588	81.9681	81.9774	81.9866	81.9958	82.0049	82.0141	82.0232	82.0322	82.0413
117	82.0503	82.0592	82.0682	82.0771	82.0860	82.0948	82.1036	82.1124	82.1212	82.1299
118	82.1386	82.1473	82.1559	82.1645	82.1731	82.1817	82.1902	82.1988	82.2073	82.2157
119	82.2241	82.2325	82.2408	82.2492	82.2575	82.2657	82.2740	82.2822	82.2903	82.2985
120	82.3066	82.3147	82.3228	82.3309	82.3389	82.3469	82.3549	82.3629	82.3708	82.3787
121	82.3865	82.3944	82.4022	82.4100	82.4178	82.4255	82.4333	82.4410	82.4486	82.4563
122	82.4639	82.4715	82.4790	82.4865	82.4940	82.5015	82.5090	82.5164	82.5238	82.5312
123	82.5386	82.5459	82.5533	82.5606	82.5679	82.5751	82.5824	82.5896	82.5968	82.6040
124	82.6112	82.6183	82.6254	82.6324	82.6395	82.6465	82.6535	82.6605	82.6675	82.6744
125	82.6814	82.6883	82.6951	82.7019	82.7088	82.7156	82.7224	82.7291	82.7359	82.7427
126	82.7494	82.7561	82.7627	82.7694	82.7760	82.7826	82.7892	82.7957	82.8023	82.8088
127	82.8153	82.8218	82.8283	82.8348	82.8412	82.8477	82.8541	82.8604	82.8668	82.8731
128	82.8794	82.8857	82.8920	82.8983	82.9045	82.9107	82.9169	82.9231	82.9292	82.9354
129	82.9415	82.9477	82.9538	82.9599	82.9660	82.9720	82.9780	82.9840	82.9900	82.9960
130	83.0019	83.0079	83.0138	83.0197	83.0256	83.0315	83.0373	83.0432	83.0490	83.0548
131	83.0606	83.0664	83.0721	83.0779	83.0836	83.0893	83.0950	83.1007	83.1063	83.1119
132	83.1176	83.1232	83.1288	83.1344	83.1400	83.1455	83.1511	83.1566	83.1621	83.1676
133	83.1730	83.1785	83.1840	83.1894	83.1949	83.2003	83.2057	83.2110	83.2164	83.2217
134	83.2271	83.2324	83.2377	83.2430	83.2483	83.2536	83.2588	83.2641	83.2693	83.2745
135	83.2797	83.2849	83.2900	83.2951	83.3003	83.3054	83.3105	83.3156	83.3207	83.3257
136	83.3308	83.3359	83.3409	83.3459	83.3509	83.3560	83.3609	83.3659	83.3709	83.3759
137	83.3808	83.3857	83.3906	83.3955	83.4004	83.4053	83.4101	83.4150	83.4198	83.4247
138	83.4295	83.4343	83.4391	83.4438	83.4486	83.4533	83.4581	83.4628	83.4676	83.4723
139	83.4770	83.4817	83.4863	83.4910	83.4956	83.5003	83.5049	83.5095	83.5141	83.5187
140	83.5233	83.5279	83.5325	83.5371	83.5417	83.5462	83.5507	83.5553	83.5598	83.5642
141	83.5687	83.5732	83.5777	83.5821	83.5866	83.5910	83.5954	83.5999	83.6043	83.6087
142	83.6130	83.6174	83.6218	83.6261	83.6305	83.6348	83.6392	83.6435	83.6478	83.6522
143	83.6565	83.6607	83.6650	83.6693	83.6735	83.6778	83.6820	83.6862	83.6904	83.6946
144	83.6988	83.7030	83.7072	83.7114	83.7156	83.7197	83.7239	83.7280	83.7321	83.7362
145	83.7403	83.7444	83.7485	83.7526	83.7567	83.7608	83.7649	83.7689	83.7730	83.7770
146	83.7810	83.7850	83.7891	83.7930	83.7970	83.8010	83.8050	83.8090	83.8130	83.8170
147	83.8209	83.8249	83.8288	83.8327	83.8366	83.8406	83.8445	83.8484	83.8522	83.8561
148	83.8600	83.8639	83.8677	83.8715	83.8754	83.8792	83.8830	83.8869	83.8907	83.8945
149	83.8983	83.9021	83.9059	83.9096	83.9134	83.9172	83.9209	83.9247	83.9285	83.9322
150	83.9359	83.9396	83.9433	83.9470	83.9507	83.9544	83.9581	83.9617	83.9654	83.9691
151	83.9727	83.9764	83.9800	83.9837	83.9873	83.9909	83.9946	83.9982	84.0018	84.0054
152	84.0090	84.0126	84.0161	84.0197	84.0233	84.0269	84.0304	84.0340	84.0375	84.0410
153	84.0446	84.0481	84.0516	84.0551	84.0587	84.0622	84.0657	84.0692	84.0727	84.0762
154	84.0796	84.0831	84.0866	84.0900	84.0935	84.0969	84.1004	84.1038	84.1072	84.1106
155	84.1140	84.1174	84.1208	84.1242	84.1276	84.1310	84.1344	84.1378	84.1412	84.1445
156	84.1479	84.1513	84.1546	84.1579	84.1613	84.1646	84.1679	84.1713	84.1746	84.1779
157	84.1812	84.1845	84.1878	84.1911	84.1943	84.1976	84.2009	84.2041	84.2074	84.2107
158	84.2139	84.2172	84.2204	84.2237	84.2269	84.2301	84.2333	84.2366	84.2398	84.2430
159	84.2461	84.2493	84.2525	84.2557	84.2588	84.2620	84.2652	84.2683	84.2715	84.2746

TABLE IV.—*Continued.*

Inclination and velocity table; $\frac{d^2}{w} D = D_{V_1} - D_{V_2}$.

Ogival-headed projectiles, 1½ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
160	84.2778	84.2809	84.2840	84.2871	84.2902	84.2933	84.2965	84.2996	84.3027	84.3058
161	84.3088	84.3119	84.3150	84.3180	84.3210	84.3242	84.3272	84.3302	84.3333	84.3363
162	84.3394	84.3424	84.3454	84.3484	84.3514	84.3544	84.3574	84.3604	84.3634	84.3664
163	84.3694	84.3724	84.3753	84.3783	84.3813	84.3843	84.3872	84.3902	84.3931	84.3960
164	84.3990	84.4019	84.4048	84.4078	84.4107	84.4136	84.4165	84.4194	84.4223	84.4252
165	84.4281	84.4310	84.4339	84.4367	94.4396	84.4425	84.4453	84.4482	84.4510	84.4539
166	84.4567	84.4595	84.4624	84.4652	84.4680	84.4709	84.4737	84.4765	84.4793	84.4821
167	84.4849	84.4877	84.4905	84.4933	84.4961	84.4988	84.5016	84.5044	84.5070	84.5099
168	84.5127	84.5154	84.5181	84.5209	84.5236	84.5263	84.5291	84.5318	84.5345	84.5372
169	84.5399	84.5426	84.5453	84.5480	84.5508	84.5534	84.5561	84.5588	84.5615	84.5641
170	84.5668	84.5695	84.5721	84.5748	84.5775	84.5801	84.5828	84.5854	84.5880	84.5907
171	84.5933	84.5959	84.5985	84.6012	84.6038	84.6064	84.6090	84.6116	84.6142	84.6168
172	84.6193	84.6219	84.6245	84.6271	84.6297	84.6322	84.6348	84.6373	84.6399	84.6424
173	84.6449	84.6475	84.6500	84.6525	84.6550	84.6575	84.6601	84.6626	84.6651	84.6676
174	84.6701	84.6726	84.6750	84.6776	84.6800	84.6825	84.6850	84.6875	84.6899	84.6924
175	84.6948	84.6973	84.6997	84.7022	84.7046	84.7071	84.7095	84.7119	84.7144	84.7168
176	84.7192	84.7216	84.7240	84.7264	84.7288	84.7312	84.7336	84.7360	84.7384	84.7408
177	84.7432	84.7455	84.7479	84.7503	84.7526	84.7550	84.7574	84.7597	84.7621	84.7645
178	84.7668	84.7692	84.7715	84.7739	84.7762	84.7785	84.7809	84.7832	84.7855	84.7878
179	84.7902	84.7925	84.7948	84.7972	84.7994	84.8017	84.8040	84.8063	84.8086	84.8109
180	84.8131	84.8154	84.8177	84.8199	84.8222	84.8244	84.8267	84.8289	84.8312	84.8334
181	84.8357	84.8379	84.8401	84.8424	84.8446	84.8468	84.8490	84.8513	84.8535	84.8557
182	84.8579	84.8601	84.8623	84.8645	84.8667	84.8689	84.8711	84.8732	84.8754	84.8776
183	84.8798	84.8819	84.8841	84.8863	84.8884	84.8906	84.8927	84.8949	84.8970	84.8992
184	84.9013	84.9035	84.9056	84.9077	84.9099	84.9120	84.9141	84.9162	84.9184	84.9205
185	84.9226	84.9247	84.9268	84.9289	84.9310	84.9331	84.9351	84.9372	84.9393	84.9414
186	84.9435	84.9456	84.9476	84.9497	84.9518	84.9538	84.9559	84.9580	84.9600	84.9621
187	84.9641	84.9662	84.9682	84.9702	84.9723	84.9743	84.9763	84.9784	84.9804	84.9820
188	84.9845	84.9865	84.9885	84.9905	84.9925	84.9946	84.9966	84.9986	85.0006	85.0026
189	85.0045	85.0065	85.0085	85.0105	85.0125	85.0145	85.0165	85.0185	85.0204	85.0224
190	85.0244	85.0263	85.0283	85.0303	85.0322	85.0342	85.0361	85.0380	85.0400	85.0419
191	85.0438	85.0458	85.0477	85.0496	85.0515	85.0535	85.0554	85.0573	85.0592	85.0611
192	85.0630	85.0650	85.0669	85.0687	85.0706	85.0725	85.0744	85.0763	85.0782	85.0801
193	85.0820	85.0838	85.0857	85.0876	85.0895	85.0913	85.0932	85.0951	85.0969	85.0988
194	85.1006	85.1025	85.1043	85.1062	85.1080	85.1099	85.1117	85.1136	85.1154	85.1172
195	85.1190	85.1208	85.1227	85.1245	85.1263	85.1281	85.1299	85.1317	85.1335	85.1353
196	85.1371	85.1389	85.1407	85.1425	85.1443	85.1460	85.1478	85.1496	85.1514	85.1531
197	85.1549	85.1567	85.1584	85.1602	85.1619	85.1637	85.1654	85.1672	85.1689	85.1707
198	85.1724	85.1741	85.1759	85.1776	85.1793	85.1810	85.1827	85.1844	85.1862	85.1879
199	85.1896	85.1913	85.1930	85.1947	85.1964	85.1981	85.1998	85.2014	85.2031	85.2048
200	85.2065	85.2081	85.2098	85.2115	85.2131	85.2148	85.2165	85.2181	85.2198	85.2214
201	85.2231	85.2247	85.2264	85.2280	85.2296	85.2313	85.2329	85.2346	85.2362	85.2378
202	85.2394	85.2411	85.2427	85.2443	85.2459	85.2476	85.2492	85.2507	85.2524	85.2540
203	85.2556	85.2572	85.2588	85.2604	85.2620	85.2635	85.2651	85.2667	85.2682	85.2698
204	85.2714	85.2729	85.2745	85.2760	85.2776	85.2791	85.2807	85.2822	85.2838	85.2853
205	85.2868	85.2884	85.2899	85.2915	85.2930	85.2945	85.2960	85.2975	85.2990	85.3005
206	85.3020	85.3035	85.3051	85.3066	85.3081	85.3095	85.3110	85.3125	85.3140	85.3155
207	85.3170	85.3184	85.3199	85.3214	85.3229	85.3244	85.3258	85.3273	85.3287	85.3302
208	85.3316	85.3331	85.3345	85.3360	85.3373	85.3388	85.3403	85.3417	85.3431	85.3446
209	85.3460	85.3474	85.3488	85.3503	85.3517	85.3531	85.3545	85.3559	85.3573	85.3587
210	85.3601	85.3615	85.3629	85.3643	85.3657	85.3671	85.3685	85.3698	85.3712	85.3726
211	85.3740	85.3754	85.3767	85.3781	85.3795	85.3808	85.3822	85.3836	85.3849	85.3863
212	85.3876	85.3890	85.3903	85.3917	85.3930	85.3943	85.3957	85.3970	85.3983	85.3996
213	85.4010	85.4023	85.4036	85.4049	85.4063	85.4076	85.4089	85.4102	85.4115	85.4128
214	85.4141	85.4154	85.4167	85.4180	85.4913	85.4206	85.4219	85.4232	85.4245	85.4258
215	85.4271	85.4284	85.4297	85.4309	85.4322	85.4335	85.4348	85.4360	85.4373	85.4385
216	85.4398	85.4411	85.4423	85.4436	85.4448	85.4461	85.4473	85.4485	85.4498	85.4510
217	85.4523	85.4535	85.4547	85.4560	85.4572	85.4584	85.4597	85.4609	85.4621	85.4633
218	85.4645	85.4658	85.4670	85.4682	85.4694	85.4706	85.4718	85.4730	85.4742	85.4754
219	85.4766	85.4778	85.4790	85.4802	85.4814	85.4825	85.4837	85.4849	85.4861	85.4873

TABLE IV.—*Continued.*

Inclination and velocity table; $\frac{d^2}{w} D = D_{V_1} - D_{V_2}$.

Ogival-headed projectiles, $1\frac{1}{2}$ diameter heads.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
220	85.4885	85.4896	85.4908	85.4920	85.4932	85.4943	85.4955	85.4967	85.4978	85.4990
221	85.5001	85.5013	85.5024	85.5036	85.5047	85.5059	85.5070	85.5082	85.5093	85.5105
222	85.5116	85.5128	85.5139	85.5150	85.5162	85.5173	85.5184	85.5195	85.5207	85.5218
223	85.5229	85.5240	85.5251	85.5262	85.5273	85.5285	85.5296	85.5307	85.5318	85.5329
224	85.5340	85.5351	85.5362	85.5373	85.5384	85.5394	85.5405	85.5416	85.5427	85.5438
225	85.5449	85.5460	85.5470	85.5481	85.5492	85.5502	85.5513	85.5524	85.5534	85.5545
226	85.5556	85.5566	85.5577	85.5588	85.5598	85.5609	85.5619	85.5630	85.5640	85.5651
227	85.5661	85.5672	85.5682	85.5693	85.5703	85.5713	85.5724	85.5734	85.5744	85.5755
228	85.5765	85.5775	85.5785	85.5796	85.5806	85.5816	85.5826	85.5836	85.5846	85.5856
229	85.5866	85.5876	85.5886	85.5896	85.5906	85.5916	85.5926	85.5936	85.5946	85.5956
230	85.5966	85.5976	85.5986	85.5996	85.6006	85.6015	85.6025	85.6035	85.6045	85.6055
231	85.6064	85.6074	85.6084	85.6094	85.6103	85.6113	85.6123	85.6132	85.6142	85.6151
232	85.6161	85.6171	85.6180	85.6190	85.6199	85.6209	85.6218	85.6228	85.6237	85.6247
233	85.6256	85.6265	85.6275	85.6284	85.6294	85.6303	85.6312	85.6321	85.6331	85.6340
234	85.6349	85.6358	85.6367	85.6377	85.6386	85.6395	85.6404	85.6413	85.6422	85.6431
235	85.6441	85.6450	85.6459	85.6468	85.6477	85.6486	85.6495	85.6504	85.6513	85.6522
236	85.6531	85.6540	85.6549	85.6558	85.6566	85.6575	85.6584	85.6593	85.6602	85.6611
237	85.6619	85.6628	85.6637	85.6646	85.6654	85.6663	85.6672	85.6680	85.6689	85.6698
238	85.6706	85.6715	85.6724	85.6732	85.6741	85.6749	85.6758	85.6766	85.6775	85.6783
239	85.6792	85.6800	85.6809	85.6817	85.6826	85.6834	85.6843	85.6851	85.6859	85.6868
240	85.6876	85.6885	85.6893	85.6901	85.6909	85.6918	85.6926	85.6934	85.6942	85.6951
241	85.6959	85.6967	85.6975	85.6984	85.6992	85.7000	85.7008	85.7016	85.7024	85.7032
242	85.7041	85.7049	85.7057	85.7065	85.7073	85.7081	85.7089	85.7097	85.7105	85.7113
243	85.7121	85.7128	85.7136	85.7144	85.7152	85.7160	85.7168	85.7176	85.7184	85.7192
244	85.7200	85.7207	85.7215	85.7223	85.7231	85.7239	85.7246	85.7254	85.7262	85.7270
245	85.7277	85.7285	85.7293	85.7301	85.7308	85.7316	85.7324	85.7331	85.7339	85.7346
246	85.7354	85.7362	85.7369	85.7377	85.7384	85.7392	85.7399	85.7407	85.7414	85.7422
247	85.7429	85.7436	85.7444	85.7451	85.7459	85.7466	85.7474	85.7481	85.7488	85.7496
248	85.7503	85.7510	85.7517	85.7525	85.7532	85.7539	85.7547	85.7554	85.7561	85.7568
249	85.7575	85.7583	85.7590	85.7597	85.7604	85.7611	85.7618	85.7626	85.7633	85.7640

TABLE V.

Showing the Resistance of the Air in pounds to Ogival-headed Projectiles of 1½ diameters' radius, from 1 to 20 inches in diameter, for Specified Velocities.

Velocity.	Diameter.				
	1 in.	5 in.	10 in.	15 in.	20 in.
f. s. 100	lbs. 0.018	lbs. 0.5	lbs. 1.8	lbs. 4.1	lbs. 7.2
500	0.473	12.0	47.0	106.0	189.0
1000	2.329	58.0	233.0	524.0	932.0
1500	10.263	257.0	1026.0	2309.0	4105.0
2000	17.096	427.0	1710.0	3847.0	6838.0
2500	26.406	660.0	2641.0	5941.0	10562.0
2800	35.452	886.0	3545.0	7977.0	14181.0